XCSP3
An Integrated Format for Benchmarking Combinatorial Constrained Problems

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XCSP3 website: www.xcsp.org
Abstract

We propose a major revision of the format XCSP 2.1, called XCSP3, to build integrated representations of combinatorial constrained problems. This new format is able to deal with mono/multi optimization, many types of variables, cost functions, reification, views, annotations, variable quantification, distributed, probabilistic and qualitative reasoning. The new format is made compact, highly readable, and rather easy to parse. Interestingly, it captures the structure of the problem models, through the possibilities of declaring arrays of variables, and identifying syntactic and semantic groups of constraints. The number of constraints is kept under control by introducing a limited set of basic constraint forms, and producing almost automatically some of their variations through lifting, restriction, sliding, logical combination and relaxation mechanisms. As a result, XCSP3 encompasses practically all constraints that can be found in major constraint solvers developed by the CP community. A website, which is developed conjointly with the format, contains many models and series of instances. The user can make sophisticated queries for selecting instances from very precise criteria. The objective of XCSP3 is to ease the effort required to test and compare different algorithms by providing a common test-bed of combinatorial constrained instances.
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Introduction

Constraint Programming (CP) suffers from the lack of a standard format for representing various forms of combinatorial problems subject to constraints to be satisfied and objectives to be optimized. Here, we refer to the possibility of generating and exchanging files containing precise descriptions of problem instances, so that fair comparisons of problem-solving approaches can be made in good conditions, and experiments can be reproduced easily. In this paper, we present the specifications for a major revision/extension of the basic format XCSP 2.1, which was introduced in 2009. This new format, XCSP3, looks like a light language that allows us to build integrated representations of combinatorial constrained problems, by enumerating variables, constraints and objectives in a very simple, structured and unambiguous way. XCSP3 can be seen as an intermediate CP format preserving the structure of problem models. In other words, XCSP3 is neither a flat format, such as XCSP 2.1 or FlatZinc, nor a modeling language such as those dedicated to mathematical programming - e.g. AMPL (http://www.ampl.com) and GAMS (http://www.gams.com) - or those dedicated to constraint programming\(^1\) - e.g. OPL [132], EaCL [130], NCL [145], ESRA [53], Zinc [42] and ESSENCE [55]. Interestingly, it is now possible to use a Java-based API, called MCSP3 [78], for modeling constrained problems and compiling them into XCSP3 instances.

As a solid basis for XCSP3, we employ the Extensible Markup Language (XML) [140], which is a flexible text format playing an important role in the exchange of a wide variety of data on the Web. At this point, it is important to note that XML is only here to support the overall architecture of our language, facilitating the issue of parsing files through common tools such as DOM (Document Object Model) and SAX (Simple API for XML), and permitting possible transformations using XSLT (Extensible Stylesheet Language Transformations). Within the global architecture, there are various XML elements that are not totally described in XML terms (for example, each integer is not put within its own element). This choice we deliberately made allows compact, readable and easily modifiable problem representations, without burdening too much on parser developers because internal data that need to be analyzed are quite

\(^1\)Note that the specification language Z (http://vl.users.org) has also been used to build nice (high-level) problem models [121]
basic. In other words, XCSP3 aims at being a good compromise between simplicity, readability and structure.

1.1 Features of XCSP3

Actually, here are the main advantages of XCSP3:

- **Large Range of Frameworks.** XCSP3 allows us to represent many forms of combinatorial constrained problems since it can deal with:
  - constraint satisfaction: CSP (Constraint Satisfaction Problem)
  - mono and multi-objective optimization: COP (Constraint Optimization Problem)
  - preferences and costs: WCSP (Weighted Constraint Satisfaction Problem) and FCSP (Fuzzy Constraint Satisfaction Problem)
  - variable quantification: QCSP(+) (Quantified Constraint Satisfaction Problem) and QCOP(+) (Quantified Constraint Optimization Problem)
  - probabilistic constraint reasoning: SCSP (Stochastic Constraint Satisfaction Problem) and SCOP (Stochastic Constraint Optimization Problem)
  - qualitative reasoning: QSTR (Qualitative Spatial and Temporal Reasoning) and TCSP (Temporal Constraint Satisfaction Problem)
  - continuous constraint solving: NCSP (Numerical Constraint Satisfaction Problem) and NCOP (Numerical Constraint Optimization Problem)
  - distributed constraint reasoning: DisCSP (Distributed Constraint Satisfaction Problem) and DCOP (Distributed Constraint Optimization Problem)

- **Large Range of Constraints.** A very large range of constraints is available, encompassing practically (i.e., to a very large extent) all constraints that can be found in major constraint solvers such as, e.g., Choco and Gecode.

- **Limited Number of Concepts.** We paid attention to control the number of concepts and basic constraint forms, advisedly exploiting automatic variations of these forms through lifting, restriction, sliding, combination and relaxation mechanisms, so as to facilitate global understanding.

- **Readability.** The new format is more compact, and less redundant, than XCSP 2.1, making it very easy to read and understand, especially as variables and constraints can be handled under the form of arrays and groups. Also, anyone can modify very easily an instance by hand. We do believe that a rookie in CP can easily manage (read and write) small instances in XCSP3.

- **Flexibility.** It will be very easy to extend the format, if necessary, in the future, for example by adding new kind of global constraints, or by adding a few XML attributes in order to handle new concepts.

- **Ease of Parsing.** Thanks to the XML architecture of the format, basically, it is easy to parse instance files at a coarse-grain level. Besides, parsers written in Java and C++ are available.

- **Dedicated Website.** A website, companion of XCSP3, is available, with many downloadable models/series/instances. This website allows the user to make (possibly sophisticated) queries in order to select and download the instances that he finds relevant.
Before starting the description of the new format, let us briefly introduce what are the main novelties of XCSP3 with respect to XCSP 2.1:

- **Optimization.** XCSP3 can manage both mono-objective and multi-objective optimization.
- **New Types of Variables.** It is possible to define 0/1, integer, symbolic, real, stochastic, set, qualitative, and graph variables, in XCSP3.
- **Lifted and Restricted forms of Constraints.** It is natural to extend basic forms of constraints over lists (tuples), sets and multi-sets. It is simple to build restricted forms of constraints by considering some properties of lists.
- **Meta-constraints.** It is possible to exploit sliding and logical mechanisms over variables and constraints.
- **Soft constraints.** Cost-based relaxed constraints and cost functions can be defined easily.
- **Reification.** Half and full reification is easy, and made possible by letting the user associate a 0/1 variable with any constraint of the problem through a dedicated XML attribute.
- **Views.** In XCSP3, it is possible to post constraints with arguments that are not limited to simple variables or constants, thus, avoiding in some situations the necessity of introducing, at modeling time, auxiliary variables and constraints, and permitting solvers that can handle variable views to do it.
- **Preservation of Structure.** It is possible to post variables under the form of arrays (of any dimension) and to post constraints in (semantic or syntactic) groups, thereby preserving the structure of the models. As already mentioned, XCSP3 is said to be an intermediate format (and not a flat one).
- **No Redundancy.** In XCSP3, redundancy is very limited, making instance representations more compact and less error-prone.
- **Compactness.** Posting variables and constraints in arrays/groups, while avoiding redundancy, makes instances in XCSP3 more compact than similar instances in XCSP 2.1.
- **Annotations.** It is possible to add annotations to the instances, for indicating for example search guidance (heuristics) and filtering advices.

As you may certainly imagine from this description, XCSP3 is a major rethinking of XCSP 2.1. Because some problems inherent to the way XCSP 2.1 was developed had to be fixed, we have chosen not to make XCSP3 backward-compatible. However, do not feel concerned too much about this issue because all series of instances available in XCSP 2.1 are translated into XCSP3, and as said above, parsers are made available.

### 1.2 Complete Modeling Chain

It is very important to understand that XCSP3 is not a modeling language: XCSP3 aims at being a rich format, simple to be used by both the human and the machine (parser/solver). We show this graphically in Figure 1.1. XCSP3 can be considered as an intermediate format.
Figure 1.1: Modeling Languages, Intermediate and Flat Formats.

Figure 1.2: Complete chain of production for modeling and solving combinatorial constrained problems, based on MCSP3 and XCSP3.
It is not flat as FlatZinc and XCSP 2.1 can be, since XCSP3 allows us to specify arrays of variables and groups/blocks of constraints.

To make things clear, do understand that the representation of constrained problems can be handled at three different levels of abstraction:

- **high level of abstraction.** At this level, modeling languages are used. They permit to use control structures (such as loops) and separate models from data. It means that for a given problem, you typically write a model which is a kind of problem abstraction parameterized by some formal data. For a specific instance, you need to provide the effective data. In practice, you have a file for the model, and for each instance you have an additional file containing the data.

- **intermediate level.** At this level, there is no separation between the model and the data, although the structure remains visible. XCSP3 is the only intermediate format proposed in the literature. For each instance, you only have a file.

- **low level.** At this level, we find flat formats. Each component (variable or constraint) of a problem instance is represented independently. As a consequence, the initial structure of the problem is lost.

To summarize, our team now proposes a complete chain of production for solving combinatorial constrained problems. The two main ingredients are:

- **MCSP3:** a Java-based API for modeling constrained problems in a natural and declarative way. Currently, this is a release candidate of the API (with its compiler). It is available on github: https://github.com/xcsp3team/XCSP3-Java-Tools, in package modeler.

- **XCSP3:** an intermediate format used to represent problem instances while preserving structure.

As shown in Figure 1.2, the user has to:

- write a model using the Java modeling API called MCSP3
- provide the data (in JSON format) for some specific instances
- compile model and data in order to get XCSP3 files
- solve the XCSP3 instance(s) using solvers like for example Choco or OscaR

The complete chain of production, called MCSP3-XCSP3, has many advantages:

- JSON, Java and XML are robust mainstream technologies
- Using JSON for data permits to have a unified notation, easy to read for both humans and machines
- using Java 8 for modeling permits the user to avoid learning again a new programming language
- Using a coarse-grained XML structure permits to have quite readable problem descriptions, easy to read for both humans and machines

**Remark 1** Using JCSP3 for representing instances, i.e., JSON instead of XML, is possible but has some (minor) drawbacks, as explained in an appendix of this document.

In the following sections of this chapter, we provide some basic information concerning the syntax and the semantics of XCSP3, which will be useful for reading the rest of the document. More precisely, we present the skeleton of typical XCSP3 instances, and introduce a few important notes related to XCSP3 syntax and semantics.
1.3 Skeleton of XCSP3 Problem Instances

To begin with, below, you will find the syntactic form of the skeleton of a typical XCSP3 problem instance, i.e., the skeleton used by most\textsuperscript{2} of the frameworks recognized by XCSP3. In our syntax, as shown in Appendix B, Page 209, we combine XML and BNF. XML is used for describing the main elements and attributes of XCSP3, whereas BNF is typically used for describing the textual contents of XCSP3 elements and attributes, with in particular \[ \] , ( ), * and + respectively standing for alternation, optional character, grouping and repetitions (0 or more, and 1 or more). Note that \textless elt.../\textgreater denotes an XML element of name elt whose description is given later in the document (this is the meaning of ...). Strings written in dark blue italic form refer to BNF non-terminals defined in Appendix B. For example, \textit{frameworkType} corresponds to a token chosen among CSP, COP, WCSP, ... Also, for the sake of simplicity, we write \textless constraint.../\textgreater and \textless metaConstraint.../\textgreater for respectively denoting any constraint element such as e.g., \textless extension.../\textgreater or \textless allDifferent.../\textgreater, and any meta-constraint such as e.g., \textless slide.../\textgreater or \textless not.../\textgreater.

```
Syntax 1
<instance format="XCSP3" type="frameworkType">
  <variables>
    ( <var.../>
    | <array.../> )+
  </variables>
  <constraints>
    ( <constraint.../>
    | <metaConstraint.../>
    | <group.../> |
    | <block.../> )*
  </constraints>
  [<objectives [combination="combinationType"]>
    ( <minimize.../>
    | <maximize.../>)+
  ]
  [<annotations.../>]
</instance>
```

As you can observe, a typical instance is composed of variables, constraints, and possibly objectives (and annotations). Variables are declared stand-alone (element \textless var.../\textgreater) or under the form of arrays (element \textless array.../\textgreater). Constraints can be elementary (element \textless constraint.../\textgreater or more complex (element \textless metaConstraint.../\textgreater). Constraints can also be posted in groups and/or declared in blocks; \textless group.../\textgreater and \textless block.../\textgreater correspond to structural mechanisms. Finally, any objective boils down to minimizing or maximizing a certain function, and is represented by an element \textless minimize.../\textgreater or \textless maximize.../\textgreater.

Remark 2 \textit{It is important to note that} \textless objectives\textgreater and \textless annotations\textgreater \textit{are optional.}

\textsuperscript{2}Syntactic details for various frameworks are given in Chapter 11.
For a first illustration of XCSP3, let us consider the small constraint network depicted (by its compatibility graph) in Figure 1.3. We have here three symbolic variables, $x$, $y$, $z$ of domain \{a, b\} and three binary constraints, one per pair of variables. An edge in this graph means a compatibility between two values of two different variables. In XCSP3, we obtain:

**Example 1**

```xml
<instance format="XCSP3" type="CSP">
    <variables>
        <var id="x" type="symbolic"> a b </var>
        <var id="y" type="symbolic"> a b </var>
        <var id="z" type="symbolic"> a b </var>
    </variables>
    <constraints>
        <extension>
            <list> x y </list>
            <supports> (a,a)(b,b) </supports>
        </extension>
        <extension>
            <list> x z </list>
            <supports> (a,a)(b,b) </supports>
        </extension>
        <extension>
            <list> y z </list>
            <supports> (a,b)(b,a) </supports>
        </extension>
    </constraints>
</instance>
```

Figure 1.3: A toy constraint network

Of course, we shall give all details of this representation in the next chapters, but certainly you are comfortable to make the correspondence between the figure and the XCSP3 code. If
you pay attention to the constraints, you can detect that two of them are similar. Indeed, the list of supports (compatible pairs) between $x$ and $y$ is the same as that between $x$ and $z$. In XCSP3, it is possible to avoid such redundancy by using either the concept of constraint group or the attribute as. Also, because the three variables share the same type and the same domain, it is possible to avoid redundancy by using either the concept of variable array or the attribute as. We shall describe all these concepts in the next chapters.

As a second illustration, we consider the arithmetic optimization example introduced in the MiniZinc Tutorial: “A banana cake which takes 250g of self-raising flour, 2 mashed bananas, 75g sugar and 100g of butter, and a chocolate cake which takes 200g of self-raising flour, 75g of cocoa, 150g sugar and 150g of butter. We can sell a chocolate cake for $4.50 and a banana cake for $4.00. And we have 4kg self-raising flour, 6 bananas, 2kg of sugar, 500g of butter and 500g of cocoa. The question is how many of each sort of cake should we bake for the fete to maximize the profit?” Here is how this small problem can be encoded in XCSP3. Surely, you can analyze this piece of code (if you are told that le stands for “less than or equal to”). Notice that any XCSP3 element can be given the optional attribute note, which is used as a short comment.

**Example 2**

```xml
<instance format="XCSP3" type="COP">  
<variables>  
  <var id="b" note="number of banana cakes"> 0..100 </var>  
  <var id="c" note="number of chocolate cakes"> 0..100 </var>  
</variables>  
<constraints>  
  <intension> le(add(mul(250,b),mul(200,c)),4000) </intension>  
  <intension> le(mul(2,b),6) </intension>  
  <intension> le(add(mul(75,b),mul(150,c)),2000) </intension>  
  <intension> le(add(mul(100,b),mul(150,c)),500) </intension>  
  <intension> le(mul(75,c),500) </intension>  
</constraints>  
<objectives>  
  <maximize> add(mul(b,400),mul(c,450)) </maximize>  
</objectives>  
</instance>
```

As we shall see, we can group together the constraints that share the same syntax (by introducing so-called constraint templates, which are abstract forms of constraints with parameters written %i) and we can describe specialized forms of objectives. An equivalent representation of the previous instance is:

**Example 3**

```xml
<instance format="XCSP3" type="COP">  
<variables>  
  <var id="b" note="number of banana cakes"> 0..99 </var>  
  <var id="c" note="number of chocolate cakes"> 0..99 </var>  
</variables>  
<constraints>  
  <group>  
  </group>  
</constraints>  
</instance>
```
Alternatively, we could even use the global constraint \( \text{sum} \), so as to obtain:

Example 4

\[
\begin{align*}
\text{<instance format} & = \text{XCSP3 type} = \text{COP}> \\
\text{<variables}> \\
\text{<var id} & = \text{b} \text{ note} = \text{number of banana cakes} > \ 0 \ldots 99 \text{ </var}> \\
\text{<var id} & = \text{c} \text{ note} = \text{number of chocolate cakes} > \ 0 \ldots 99 \text{ </var}> \\
\text{</variables}> \\
\text{<constraints}> \\
\text{<sum note} = \text{using the 4000 grams of flour}> \\
\text{<list> b c </list}> \\
\text{<coeffs> 250 200 </coeffs> } \\
\text{<condition> (le,4000) </condition> } \\
\text{</sum> } \\
\text{<sum note} = \text{using the 6 bananas}> \\
\text{<list> b </list> } \\
\text{<coeffs> 2 </coeffs> } \\
\text{<condition> (le,6) </condition> } \\
\text{</sum> } \\
\text{<sum note} = \text{using the 2000 grams of sugar}> \\
\text{<list> b c </list> } \\
\text{<coeffs> 75 150 </coeffs> } \\
\text{<condition> (le,2000) </condition> } \\
\text{</sum> } \\
\text{<sum note} = \text{using the 500 grams of butter}> \\
\text{<list> b c </list> } \\
\text{<coeffs> 100 150 </coeffs> } \\
\text{<condition> (le,500) </condition> } \\
\text{</sum> } \\
\text{<sum note} = \text{using the 500 grams of cocoa}> \\
\text{<list> c </list> }
\end{align*}
\]
Since we have just mentioned MiniZinc, let us claim again that XCSP3 is not a modeling language. XCSP3 has no advanced control structures and no mechanism for separating model and data, but aims at being an intermediate format, simple to be used by both the human and the machine (parser/solver) while keeping the structure of the problems.

### 1.4 XML versus JSON

For various reasons, some people prefer working with JSON, instead of XML. As shown in Appendix D, we can easily convert XCSP3 instances from XML to JSON by following a few rules. We illustrate this on the previous examples. Their JSON representations are shown below.

**Example 5**

```json
{
  "@format":"XCSP3",
  "@type":"CSP",
  "variables":{
    "var": [{
      "@id":"x",
      "domain":"a b"
    }, {
      "@id":"y",
      "domain":"a b"
    }, {
      "@id":"z",
      "domain":"a b"
    }]
  },
  "constraints":{
    "extension": [{
      "list":"x y",
      "supports":"(a,a)(b,b)"
    }, {
      "list":"x z",
      "supports":"(a,a)(b,b)"
    }]
  }"}
```
Example 6

```json
{
  "@format": "XCSP3",
  "@type": "COP",
  "variables": {
    "var": [
      {
        "@id": "b",
        "domain": "0..100"
      },
      {
        "@id": "c",
        "domain": "0..100"
      }
    ]
  },
  "constraints": {
    "intension": [
      "le( add( mul(250, b), mul(200, c)), 4000 )",
      "le( mul(2, b), 6 )",
      "le( add( mul(75, b), mul(150, c)), 2000 )",
      "le( add( mul(100, b), mul(150, c)), 500 )",
      "le( mul(75, c), 500 )"
    ]
  },
  "objectives": {
    "maximize": "add( mul(b, 400), mul(c, 450) )"
  }
}
```

Remark 3 Although JSON looks attractive, there exist some limitations that make us preferring XML. These limitations are discussed in Appendix D.

1.5 XCSP3-core

XCSP3-core is a subset of XCSP3, with what can be considered as the main concepts of Constraint Programming (although, of course, this is quite subjective). The interest of XCSP3-core is multiple:

- focusing on the most popular constraints,
- facilitating parsing process through dedicated parsers in Java and C++, using callback functions,
- and defining a core format for future competitions of constraint solvers.
XCSP3-core, targeted for CSP (Constraint Satisfaction Problem) and COP (Constraint Optimization problem), handles integer variables, mono-objective optimization, and 20 important constraints, described in the next chapters of this document:

- intension
- extension
- regular
- mdd
- allDifferent
- allEqual
- ordered (lex)
- sum (linear)
- count (capturing atLeast, atMost, exactly, among)
- nValues
- cardinality
- minimum
- maximum
- element
- channel
- noOverlap (capturing disjunctive and diffn)
- cumulative
- circuit
- instantiation
- slide

Importantly, as for full XCSP3, the structure of problems (models) can be preserved in XCSP3-core thanks to the following mechanisms:

- arrays of variables
- groups of constraints
- blocks of constraints
- the meta-constraint slide

More information about XCSP3-core can be found at www.xcsp.org. Finally, note that the following parsers are available:

- a Java 8 parser for XCSP3-core (and also for a large part of full XCSP3)
- a C++ parser for XCSP3-core

### 1.6 Important Notes about Syntax and Semantics

As already indicated, for the syntax, we invite the reader to consult Appendix B. In this section, we discuss some important points.
Constraint Parameters and Numerical Conditions. One important decision concerns the way constraint parameters are defined. We have adopted the following rules:

1. Most of the time, when defining a constraint, there is a main list of variables (not necessarily, its full scope) that must be handled. We have chosen to call it <list>. Identifying the main list of variables (using its type/form) permits in a very simple and natural way to derive variants of constraints, by using alternative forms such as as <set> and <mset> as shown in Chapter 7.

2. Also, quite often, we need to introduce numerical conditions (comparisons) composed of an operator \(\circ\) and a right-hand side operand \(k\) that is a value, a variable, an interval or a set; the left-hand side being indirectly defined by the constraint. The numerical condition is a kind of terminal operation to be applied after the constraint has “performed some computation”. We propose to represent each numerical condition, i.e., each pair \((\circ, k)\) by a single XCSP3 element <condition> containing both the operator and the right operand, between parentheses and with comma as separator.

3. All other parameters are given by their natural names, as for example <transitions> for the constraint regular, or <index> for the constraint element.

It is important to note that each numerical condition \((\circ, k)\) semantically stands for “\(\circ k\)”, where:

- either \(\circ \in \{<,\leq,>,\geq,=,\neq\}\) and \(k\) is a value or a variable; in XCSP3, we thus have to choose a symbol in \{lt,le,gt,ge,eq,ne\} for \(\circ\),
- or \(\circ \in \{\in,\notin\}\) and \(k\) is an integer interval of the form \(l..u\), or an integer set of the form \(\{a_1,\ldots,a_p\}\) with \(a_1,\ldots,a_p\) integer values; in XCSP3, we thus have to choose a symbol in \{in,notin\} for \(\circ\).

For example, we may have \((\circ, k)\) denoting “\(>10\)”, “\(\neq z\)” or “\(\in 1..5\)” by respectively writing \((gt,10)\), \((ne,z)\) and \((in,1..5)\). In XCSP3, we shall syntactically represent a numerical condition as follows:

```
Syntax 2
| <condition> "(" operator "," operand ")" </condition>
```

where operator and operand are two BNF non-terminals defined in Appendix B.

Attributes id, as and reifiedBy. In the next chapters, we shall present the syntax as precisely as possible. However, we shall make the following simplifications.

Any XML element in XCSP3 can be given a value for the attribute id. This attribute is required for the elements <var.../> and <array.../> but it is optional for all other elements. When presenting the syntax, we shall avoid to systematically write [id="identifier"], each time we put an element for which id is optional. Note that, as for HTML5, each value of an attribute id must be document-wide unique.

Any XML element in XCSP3 can be given a value for the attribute as. This attribute is always optional. Again, when presenting the syntax, we shall avoid to systematically write [as="identifier"], each time we introduce an element. This form of aliasing is discussed in Chapter 10.

\[\text{Exceptions are for some scheduling constraints, where <origins> is more appropriate.}\]
Any constraint and meta-constraint can be given a value for one of the three following attributes \texttt{reifiedBy}, \texttt{reifiedFrom} and \texttt{reifiedTo}. These attributes are optional and mutually exclusive. When presenting the syntax in this document, we shall avoid to systematically write \texttt{[reifiedBy="01Var"]|reifiedFrom="01Var"]|reifiedTo="01Var"]} each time we introduce a constraint or meta-constraint. Reification is discussed in Chapter 10.

\textbf{Views.} For clarity, we always give the syntax of constraints in a rather usual restricted form. For example, we shall write \texttt{(intVar wspace)*} when a list of integer variables is naturally expected. However, most of time, it is possible to replace variables by more general expressions (for example, we could have \texttt{add(x,10)} where a variable is expected). This is acceptable (the XML schema that we provide for XCSP3 allows such forms) although we employ a more restrictive syntax in this document. Views are discussed in Chapter 10.

\textbf{Whitespaces.} The tolerance with respect to whitespaces is the following:

- leading and trailing whitespaces for values of XML attributes are not tolerated. For example, \texttt{id=" x"} is not valid.
- leading and trailing whitespaces for textual contents of XML elements are tolerated, and not required. Although, for the sake of simplicity, we prefer writing:

\begin{verbatim}
  <list> (intVar wspace)* </list>
\end{verbatim}

instead of the heavier and more precise form:

\begin{verbatim}
  <list> [intVar (wspace intVar)*] </list>
\end{verbatim}

it must be clear that no trailing whitespace is actually required.
- numerical conditions must not contain any whitespace. For example, \texttt{(lt, 10)} is not valid.
- functional expressions must not contain any whitespace. For example, \texttt{add(x,y)} is not valid.
- whitespaces are tolerated between vectors (tuples) but not within vectors, where a vector (tuple) is a sequence of elements put between brackets with comma as separator. For example, \texttt{<supports> (1,3)( 2,4) </supports>} is not valid, whereas:

\begin{verbatim}
  <supports>
    (1,3)
    (2,4)
  </supports>
\end{verbatim}

is.
- whitespaces are not tolerated at all when expressing decimals, rationals, integer intervals, real intervals, and probabilities. For example, \texttt{3. 14, 3 /2, 1 .. 10, [1.2, 5]}, and \texttt{2: 1/6} are clearly not valid expressions.
Semantics. Concerning the semantics, here are a few important remarks:

- when presenting the semantics, we distinguish between a variable \( x \) and its assigned value \( x \) (note the bold face on the symbol \( x \)).

- Concerning the semantics of a numerical condition \((\circ, k)\), and depending on the form of \( k \) (a value, a variable, an interval or a set), we shall indiscriminately use \( k \) to denote the value \( k \), the value of the variable \( k \), the interval \( l..u \) represented by \( k \), or the set \( \{a_1, \ldots, a_p\} \) represented by \( k \).

1.7 Notes (Short Comments) and Classes

In XCSP3, it is possible to associate a note (short comment) with any element. It suffices to use the attribute `note` whose value can be any string. Of course, for simplicity, when presenting the syntax, we shall never write `[note="string"]`, each time an element is introduced. Here is a small instance with two variables and one constraint, each one accompanied by a note.

```xml
<instance format="XCSP3" type="CSP">
  <variables>
    <var id="x" note="x is a number between 1 and 10">1..10</var>
    <var id="y" note="y denotes the square of x">1..100</var>
  </variables>
  <constraints>
    <intension note="this constraint links x and y">
      \( \text{eq}(y, \text{mul}(x, x)) \)
    </intension>
  </constraints>
</instance>
```

In XCSP3, it is also possible to associate tags with any element. As in HTML, these tags are introduced by the attribute `class`, whose value is a sequence of identifiers, with whitespace used as a separator. Again, for simplicity, when presenting the syntax, we shall never write `[class="(identifier wspace)+"]`, each time an element is introduced. As we shall see in Section 10.2, these tags can be predefined or user-defined.

1.8 Structure of the Document

The document is organized in four parts. In the first part, we show how to define different types of variables (Chapter 2) and objectives (Chapter 3). In the second part, we describe basic forms of constraints over simple discrete variables (Chapter 4), complex discrete variables (Chapter 5) and continuous variables (Chapter 6). Advanced (i.e., non-basic) forms of constraints are presented in the third part of the document: they correspond to lifted and restricted constraints (Chapter 7), meta-constraints (Chapter 8) and soft constraints (Chapter 9). Finally, in the fourth part, we introduce groups, blocks, reification, views and aliases (Chapter 10), frameworks (Chapter 11) and annotations (Chapter 12).
Part I

Variables and Objectives
In this part, we show how variables and objectives are represented in XCSP3.
Variables are the basic components of combinatorial problems. In XCSP3, as shown in Figure 2.1, you can declare:

Figure 2.1: The different types of variables
• discrete variables, that come in three categories:
  – simple discrete variables
    * integer variables, including 0/1 variables that can be used to represent Boolean
      variables too
    * symbolic variables
  – complex discrete variables
    * set variables
    * graph variables
  – stochastic discrete variables
• continuous variables
  – real variables
  – qualitative variables

You can also declare:
• $k$-dimensional arrays of variables, with $k \geq 1$

Recall that stand-alone variables as well as arrays of variables must be put inside the element
$<variables>$, as follows:

Syntax 3

```
<variables>
  ( <var.../> | <array.../> )+
</variables>
```

Also as already mentioned, for the syntax, we combine XML and BNF, with BNF non-
terminals written in dark blue italic form (see Appendix B). The general syntax of an element
$<var>$ is:

Syntax 4

```
<var id="identifier" [type="varType"]>
  ... // Domain description
</var>
```

A stand-alone variable can thus be declared by means of an element $<var>$, and its identifica-
tion is given by the value of the required attribute $id$. In accordance with the value ("integer"
by default) of the attribute $type$, this element contains the description of its domain, i.e., the
values that can possibly be assigned to it.

Sometimes, however, an element $<var>$ has no content at all. Actually, there are three such
situations:
• when a variable is qualitative, its domain is implicit, as we shall see in Section 2.8,
• when a (non-qualitative) variable has an empty domain, as discussed in Section 2.10,
• when the attribute $as$ is present.

For the last case, we have:
The value of \texttt{as} must correspond to the value of the attribute \texttt{id} of another element. When this attribute is present, the content of the element is exactly the same as the one that is referred to (as if we applied a copy and paste operation), as discussed in Section 10.5. From now on, for simplicity, we shall systematically omit the optional attribute \texttt{as}, although its role will be illustrated in the section about integer variables.

The general syntax of an element \texttt{<array>} is:

The chapter is organized as follows. From Section 2.1 to Section 2.8, we develop the syntax for the different types of stand-alone variables, namely, 0/1 variables, integer variables, symbolic variables, real variables, set variables, graph variables, stochastic variables and qualitative variables. Then, in Section 2.9 we describe arrays of variables, and in Section 2.10 we discuss about variables with empty domains as well as about useless variables. In the last section of this chapter, Section 2.11, we show how solutions can be represented.

2.1 0/1 Variables

There is no specific language keyword for denoting a 0/1 variable. Instead, a 0/1 variable is defined as an integer variable (see next section).

\textbf{Remark 4} In XCSP3, there is no difference between a 0/1 variable and a Boolean variable: when appropriate, 0 stands for false and 1 stands for true.

2.2 Integer Variables

Integer variables are given a domain of values by listing them, using whitespace as separator. More precisely, the content of an element \texttt{<var>} that represents an integer variable is an ordered sequence of integer values and integer intervals. We have for example:

- \texttt{1 5 10} that corresponds to the set \{1, 5, 10\}.
- \texttt{1..3 7 10..14} that corresponds to the set \{1, 2, 3, 7, 10, 11, 12, 13, 14\}.
As an illustration, below, both variables foo and bar exhibit the same domain, whereas qux mixes integer and integer intervals:

**Example 8**

```xml
<var id="foo"> 0 1 2 3 4 5 6 </var>
<var id="bar"> 0..6 </var>
<var id="qux"> -6..-2 0 1..3 4 7 8..11 </var>
```

As mentioned earlier, declaring a 0/1 variable is made explicitly, i.e, with the same syntax as an “ordinary” integer variable. For example, b1 and b2 are two 0/1 variables that can be served as Boolean variables in logical expressions:

**Example 9**

```xml
<var id="b1"> 0 1 </var>
<var id="b2"> 0 1 </var>
```

Not all domains of integer variables are necessarily finite. Indeed, it is possible to use the special value *infinity*, preceded by the mandatory sign + or − (necessarily, as a bound for an integer interval). For example:

**Example 10**

```xml
<var id="x"> 0..+infinity </var>
<var id="y"> -infinity..+infinity </var>
```

Finally, when domains of certain variables are similar, and it appears that declaring array(s) is not appropriate, we can use the optional attribute as to indicate that an element has the same content as another one; the value of as must be the value of an attribute id, as explained in Section 10.5 of Chapter 10. Below, v2 is a variable with the same domain as v1.

**Example 11**

```xml
<var id="v1"> 2 5 8 9 12 15 22 25 30 50 </var>
<var id="v2" as="v1" />
```

**Remark 5** As shown by the syntax, for an integer variable, the attribute type is optional: if present, its value must be "integer".

**Remark 6** The integer values and intervals listed in the domain of an integer variable must always be in increasing order, without several occurrences of the same value. For example, 0..10 10 is forbidden to be the content of an element <var>.

### 2.3 Symbolic Variables

A symbolic variable is defined from a finite domain containing a sequence of symbols as possible values (whitespace as separator). These symbols are identifiers, and so, must start with a letter. For a symbolic variable, the attribute type for element <var> is required, and its value must be "symbolic".

27
Syntax 8

\[
\text{<var id="identifier" type="symbolic"} > \\
\text{(symbol wspace)*} \\
\text{</var>}
\]

An example is given by variables light, whose domain is \{green, orange, red\}, and person, whose domain is \{tom, oliver, john, marc\}.

Example 12

\[
\text{<var id="light" type="symbolic"} > \text{green orange red </var> \\
\text{<var id="person" type="symbolic"} > \text{tom oliver john marc </var>}
\]

Remark 7 Integer values and symbolic values cannot be mixed inside the same domain.

Remark 8 The values listed in the domain of a symbolic variable are not necessarily given in increasing (lexicographic) order, but they must be all distinct.

2.4 Real Variables

Real variables are given a domain of values by listing real intervals, using whitespace as separator. More precisely, the content of an element <var> that represents a real variable contains an ordered sequence of real intervals (two bounds separated by a comma and enclosed between open or closed square brackets), whose bounds are integer, decimal or rational values. In practice, most of the time, only one interval is given. For a real variable, the attribute type for element <var> is required, and its value must be "real".

Syntax 9

\[
\text{<var id="identifier" type="real"} > \\
\text{(realIntvl wspace)*} \\
\text{</var>}
\]

We have for example:

Example 13

\[
\text{<var id="w" type="real"} > \text{[0,+infinity[ </var> \\
\text{<var id="x" type="real"} > \text{[-4,4] </var> \\
\text{<var id="y" type="real"} > \text{[2/3,8.355] </var>}
\]

Remark 9 Whenever infinity is involved, the associated square bracket(s) must be open, so that we shall always write ]-infinity and +infinity[.

Remark 10 The intervals listed in the domain of a real variable must always be in increasing order, without overlapping. For example, \([0,10] \ [8,20]\) is forbidden as content of any element <var>.
2.5 Set Variables

Set variables have associated set domains. To begin with, we focus on integer set variables, i.e., on variables that must be assigned a set of integer values. It is usual\(^1\) that a set domain is approximated by a set interval specified by its upper and lower bounds (subset-bound representation), defined by some appropriate ordering on the domain values \([61, 62]\). The core idea is to approximate the domain of a set variable \(s\) by a closed interval denoted \([s_{\text{min}}, s_{\text{max}}]\), specified by its unique least upper bound \(s_{\text{min}}\), and unique greatest lower bound \(s_{\text{max}}\), under set inclusion. We have \(s_{\text{min}}\) that contains the required elements of \(s\) and \(s_{\text{max}}\) that contains in addition the possible elements of \(s\). For example, if \([s_{\text{min}} = \{1, 5\}, s_{\text{max}} = \{1, 3, 5, 6\}]\) is the domain of an integer set variable \(s\), then it means that the elements 1 and 5 necessarily belong to \(s\) and that 3 and 6 are possible elements of \(s\). The variable \(s\) can then be assigned any set in the lattice defined from \(s_{\text{min}}\) and \(s_{\text{max}}\).

To define an integer set variable, we just have to introduce two elements \(<\text{required}>\) and \(<\text{possible}>\) within the element \(<\var>\). Such elements contain an ordered sequence of integers and integer intervals, as for ordinary integer variables. For an integer set variable, the attribute type for \(<\var>\) is required, and its value must be "set".

**Syntax 10**

```xml
<var id="identifier" type="set">[<required> ((intVal | intIntvl) wspace)* ]<required>/ </required><possible> ((intVal | intIntvl) wspace)* </possible>]</var>
```

An integer set variable \(s\) of domain \([\{1, 5\}, \{1, 3, 5, 6\}]\) is thus represented by:

**Example 14**

```xml
<var id="s" type="set"><required> 1 5 </required><possible> 3 6 </possible></var>
```

It is also possible to define symbolic set variables. In that case, elements \(<\text{required}>\) and \(<\text{possible}>\) must each contain a sequence of symbols (identifiers). For a symbolic set variable, the attribute type for \(<\var>\) is required, and its value must be "symbolic set".

**Syntax 11**

```xml
<var id="identifier" type="symbolic set">[<required> (symbol wspace)* ]<required>/ </required><possible> (symbol wspace)* </possible>]</var>
```

The following example exhibits a symbolic set variable named \(\text{team}\), which must contain elements Bob and Paul, and can additionally contain elements Emily, Luke and Susan.

---

\(^1\)There exist alternatives to represent domains, as the length-lex representation \([63]\). We might introduce them in the future if they become more popular in solvers.
Example 15

```xml
<var id="team" type="symbolic set">
    <required> Bob Paul </required>
    <possible> Emily Luke Susan </possible>
</var>
```

Remark 11 It is forbidden to have a value present both in <required> and <possible>.

2.6 Graph Variables

Graph variables have associated graph domains. More precisely, graph variables have domains that are approximated by the lattice of graphs included between two bounds: the greatest lower bound and the least upper bound of the lattice [46]. Graph variables can be directed or undirected, meaning that either edges or arcs are handled. For a graph variable \( g \), the greatest lower bound \( g_{\text{min}} \) defines the set of nodes and edges/arcs which are known to be part of \( g \) while the least upper bound graph \( g_{\text{max}} \) defines the set of possible nodes and edges/arcs in \( g \). For example, if \( g_{\text{min}} = \{(a,b),\{(a,b)\}\}, g_{\text{max}} = \{(a,b,c),\{(a,b),(a,c),(b,c)\}\} \) is the domain of an undirected graph variable \( g \), then it means that the nodes \( a \) and \( b \), as well as the edge \( (a,b) \) necessarily belong to \( g \) and that the node \( c \) as well as the edges \( (a,c) \) and \( (b,c) \) are possible elements of \( g \).

To define a graph variable, we have to introduce two elements <required> and <possible> within the element <var>. Inside these elements, we find the elements <nodes> and <edges> for an undirected graph variable, and the elements <nodes> and <arcs> for a directed graph variable. For a graph variable, the attribute type for <var> is required, and its value must be either "undirected graph" or "directed graph".

Syntax 12

```xml
<var id="identifier" type="undirected graph">
    [<required>
        <nodes> (symbol wspace)* </nodes>
        <edges> ("(" symbol "," symbol ")")* </edges>
    </required>
    [<possible>
        <nodes> (symbol wspace)* </nodes>
        <edges> ("(" symbol "," symbol ")")* </edges>
    </possible>]
</var>
```

Syntax 13

```xml
<var id="identifier" type="directed graph">
    [<required>
        <nodes> (symbol wspace)* </nodes>
        <arcs> ("(" symbol "," symbol ")")* </edges>
    </required>
    [<possible>]
</var>
```
The example given by Figure 2.2, for an undirected graph variable $g$ whose domain is $[((a,b),\{(a,b)\}),\{(a,b,c),\{(a,b),(a,c),(b,c)\}\}]$, is thus represented by:

```
<var id="g" type="undirected graph">
  <required>
    <nodes> a b </nodes>
    <edges> (a,b) </edges>
  </required>
  <possible>
    <nodes> c </nodes>
    <edges> (a,c)(b,c) </edges>
  </possible>
</var>
```

Figure 2.2: The domain of an undirected graph variable $g$

**Remark 12** It is forbidden to have an element (node, edge, or arc) present both in `<required>` and `<possible>`.

### 2.7 Stochastic Variables

In some situations, when modeling a problem, it is useful to associate a probability distribution with the values that are present in the domain of an integer variable. Such variables are typically uncontrollable (i.e., not decision variables). This is for example the case with the Stochastic CSP (SCSP) framework introduced by T. Walsh [141] to capture combinatorial decision problems involving uncertainty. In XCSP3, the domain of an integer stochastic variable is defined as usual by a sequence of integers and integer intervals, but each element of the sequence is given a probability preceded by the symbol `:`. The value of a probability can be 0, 1, a rational value or a decimal value in $[0,1]$. Note that it is also possible to declare symbolic stochastic variables. For an integer stochastic variable, the attribute `type` for `<var>` is required, and its value must be "stochastic".
For a symbolic stochastic variable, the attribute type for <var> is required, and its value must be "symbolic stochastic".

Of course, the sum of probabilities associated with the different possible values of a stochastic variable must be equal to 1, and consequently, it is not possible to build a stochastic variable with an empty domain.

An illustration is given by:

Example 17

```
<var id="foo" type="stochastic">
  5:1/6
  15:1/3
  25:1/2
</var>
<var id="dice" type="stochastic">
  1..6:1/6
</var>
<var id="coin" type="symbolic stochastic">
  heads:0.5
  tails:0.5
</var>
```

Remark 13 When the probability is associated with an integer interval, it applies independently to every value of the interval.

2.8 Qualitative Variables

In qualitative spatial and temporal reasoning (QSTR) [72], one has to reason with entities that corresponds to points, intervals, regions, ... The variables that are introduced represent such entities, and their domains, being continuous, cannot be described extensionally. This is the reason why we simply use the attribute type to refer to the implicit domain of qualitative variables.

In XCSP3, we can currently refer to the following types:

- "point", when referring to the possible time points (or equivalently, points of the line)
- "interval", when referring to the possible time intervals (or equivalently, intervals of the line)
• "region", when referring to possible regions in Euclidean space (or in a topological space)

Syntax 16

```xml
<var id="identifier" type="point" />
<var id="identifier" type="interval" />
<var id="identifier" type="region" />
```

An illustration is given by:

Example 18

```xml
<var id="foo" type="point" />
<var id="bar" type="interval" />
<var id="qux" type="region" />
```

Remark 14 Other domains for qualitative variables might be introduced in the future.

2.9 Arrays of Variables

Interestingly, XCSP3 allows us to declare \( k \)-dimensional arrays of variables, with \( k \geq 1 \), by introducing elements `<array>` inside `<variables>`. We recall the general (simplified) syntax for arrays:

Syntax 17

```xml
<array id="identifier" [type="varType"] size="dimensions" [startIndex="integer"]>
  ...
</array>
```

Hence, for each such element, there is a required attribute `id` and a required attribute `size` whose value gives the structure of the array under the form `[nb\(_1\) \ldots nb\(_p\)]` with nb\(_1\), \ldots, nb\(_p\) being strictly positive integers. The number of dimensions of an array is the number of pairs of opening/closing square brackets, and the size of each dimension is given by its bracketed value. For example, if \( x \) is an array of 10 variables, you just write "[10]", and if \( y \) is a 2-dimensional array, 5 rows by 8 columns, you write "[5][8]". Indexing used for any dimension of an array starts at 0, unless the (optional) attribute `startIndex` gives another value. Of course, it is possible to define arrays of any kind of variables, as e.g., arrays of symbolic variables, arrays of set variables, etc. by using the attribute `type`: all variables of an array have the same type. The content of an element `<array>` of a specified type is defined similarly to the content of an element `<var>` of the same type, and basically, all variables of an array have the same domain, except if mixed domains are introduced (as we shall see in subsection 2.9.2).

To define an array \( x \) of 10 integer variables with domain 1..100, a 2-dimensional array \( y \) (5 \times 8) of 0/1 variables, an array `diceYathzee` of 5 stochastic variables, and an array \( z \) of 12 symbolic set variables with domain \([\{a,b\},\{a,b,c,d\}]\), we write:
Importantly, it is necessary to be able to identify variables in arrays. We simply use the classical “[]” notation, with indexing starting at 0 (unless another value is given by startIndex). For example, assuming that 0 is the “starting index”, \( x[0] \) is the first variable of the array \( x \), and \( y[4][7] \) the last variable of the array \( y \).

### 2.9.1 Using Compact Forms

Sometimes, one is interested in selecting some variables from an array, for example, the variables in the first row of a 2-dimensional array. We use integer intervals for that purpose, as in \( x[3..5] \) and \( y[2..3][0..1] \), and we refer to such expressions as *compact lists of array variables*. In a context where a list of variables is expected, it is possible to use this kind of notation, and the result is then considered to be a list of variables, ordered according to a lexicographic order \( < \) on index tuples (for example \( y[2][4] \) is before \( y[2][5] \) since \((2,4) < (2,5)\)). On our previous example, in a context where a list of variables is expected, \( x[3..5] \) denotes the list of variables \( x[3], x[4] \) and \( x[5] \), while \( y[2..3][0..1] \) denotes the list of variables \( y[2][0], y[2][1], y[3][0] \) and \( y[3][1] \). It is also possible to omit an index, with a meaning similar to using \( 0..s \) where \( s \) denotes the largest possible index. For example, \( y[2][] \) is equivalent to \( y[2][0..7] \).

Finally, one may wonder how compact lists of array variables (such as \( x[3..5], y[2..3][0..1], x[], y[2][] \)) precisely expand in the context of the XCSP3 elements that will be presented in the next chapters. The rule is the following:

1. if a 2-dimensional compact list (such as \( y[2][] \)) appears in an element \(<\text{matrix}>, <\text{origins}>, <\text{lengths}>, <\text{rowOCCurs}> \) or \(<\text{colOCCurs}>\), the compact list expands as a sequence of tuples (one tuple per row, with variables of the row separated by a comma, enclosed between parentheses). For example,

\[
<\text{matrix}>
\begin{array}{l}
\text{y[0][1]}
\end{array}
</\text{matrix}>
\]

expands as:

\[
<\text{matrix}>
\begin{array}{l}
y[0][0], y[0][1], \ldots, y[0][7] \\
y[1][0], y[1][1], \ldots, y[1][7] \\
\ldots
\end{array}
</\text{matrix}>
\]
2. in all other situations, the compact list expands as a list of variables, with whitespace as a separator. For example, we have:

```xml
<list>
  y[2..3][0..1]
</list>
```

that expands as:

```xml
<list>
  y[2][0] y[2][1] y[3][0] y[3][1]
</list>
```

Using this rule, it is always possible to expand all compact lists of array variables in order to get a form of the problem instance with only references to simple variables.

### 2.9.2 Dealing With Mixed Domains

Sometimes, the variables from the same array naturally have mixed domains. One solution is to build a large domain that can be suitable for all variables, and then to post unary domain constraints. But this not very satisfactory.

Another solution with XCSP3 is to insert the different definitions of domains inside the `<array>` element. When several subsets of variables of an array have different domains, we simply have to put an element `<domain>` for each of these subsets. An attribute `for` indicates the list of variables to which the domain definition applies. The value of `for` can be a sequence (whitespace as separator) of variable identifiers (with compact forms authorized), or the special value "others", which is used to declare a default domain. Only one element `<domain>` can have its attribute `for` set to "others", and if it is present, it is necessary at the last position. The syntax for arrays that involve variables with different domains is:

**Syntax 18**

```xml
<array id="identifier" [type="varType"] size="dimensions" [startIndex="integer"]>
  (<domain for="(intVar wspace)+"> ... </domain>)+
  [<domain for="others"> ... </domain>]
</array>
```

As an illustration, the 2-dimensional array `x` below is such that the variables of the first, second and third rows have `1..10`, `1..20` and `1..15` as domains, respectively. Also, all variables of array `y` have `{2, 4, 6}` as domain except for `y[4]` whose domain is `{0, 1}`. Finally, all variables of array `z` have `{0, 1}` as domain except for the variables that belong to the lists `z[0..1]` and `z[2..4]`.
2.10 Empty Domains and Useless Variables

In some (rare) cases, one may want to represent variables with empty domains. For example, if an inconsistency is detected during an inference process, it may be interesting to represent the state of the variable domains (including those that became empty). For all types of variables, except for qualitative ones, an empty content for an element <var> (or <array>) means that the domain is empty (if, of course, the attribute as is not present).

In our illustration, below, we have an integer variable \( x \) and a set variable \( y \), with both an empty domain.

#### Example 21

- \(<var id="x" />\)
- \(<var id="y" type="set" />\)

We conclude this chapter with a short discussion about the concept of undefined and useless variables. An undefined (domain) variable is a variable with no domain definition (note that this is different from a variable having an empty domain). Actually, undefined variables can only belong to arrays showing different domains for subsets of variables: when a variable does not match any value of attributes for, then it is undefined, and so must be completely ignored (by solvers).

As an illustration, below, the variable \( z[5] \) is undefined.

#### Example 22

- \(<array id="z" size="[10]" >\)
  - \(<domain for="z[0..4]">1..10</domain>\)
  - \(<domain for="z[6..9]">1..20</domain>\)
- \(<array id="z" size="[3][5]" >\)
  - \(<domain for="x[0][]">1..10</domain>\)
  - \(<domain for="x[1][]">1..20</domain>\)
  - \(<domain for="x[2][]">1..15</domain>\)
- \(<array id="y" size="[10]" >\)
  - \(<domain for="y[4]">0 1</domain>\)
  - \(<domain for="others">2 4 6</domain>\)
- \(<array id="z" size="[6][5][5]" >\)
  - \(<domain for="z[][0..1][] z[][2][2..4]">0..10</domain>\)
  - \(<domain for="others">0 1</domain>\)

A variable is said useless if it is involved nowhere (neither in constraints nor in objective functions); otherwise, it is said useful. On the one hand, a useless variable can be the result of a reformulation/simplification process (and one may wish to keep such variables, for various
reasons). On the other hand, modeling can introduce useless variables, typically for symmetrical reasons, which may happen when introducing arrays of variables.

For the example below, suppose that only variables \([i][j]\) of \(t\) such that \(i > j\) are involved in constraints. This means that all variables \([i][j]\) with \(i \leq j\) are useless.

Example 23

```xml
<array id="t" size="[8][8]"> 1..10 </array>
```

Remark 15 It is not valid to have a variable both undefined and useful. In other words, it is not permitted in XCSP3 to have an undefined variable that is involved in a constraint or an objective.

To summarize, undefined variables must be ignored by solvers, and useless variables can be discarded. We might consider introducing in the future new elements, attributes or annotations for clearly specifying which variables are useless, but note that it can be easily identified by parsers. The tool for checking solutions (and their costs) that we are currently developing will be able to handle the presence or the absence of useless variables. So, users will be given the possibility to submit complete instantiations or partial instantiations only involving useful variables.

2.11 Solutions

In XCSP3, we can also represent solutions, i.e., XCSP3 output. We simply use an element `<instantiation>` that gives a value for each (useful) variable of the instance. More precisely, to define a solution, we introduce two elements `<list>` and `<values>` inside the element `<instantiation>`: the \(i\)th variable of `<list>` is assigned the \(i\)th value of `<values>`. Obviously, it is not possible to have several occurrences of the same variable inside `<list>`.

There is a required attribute `type`. For decision problems, e.g., CSP or SCSP, its value is necessarily "solution". For optimization problems, e.g., COP or WCSP, its value is either "solution" or "optimum", and another required attribute `cost` gives the cost of the (optimal) solution. The syntax is given below for integer variables only.

Syntax 19

```xml
<instantiation type="solution|optimum" [cost="integer"] >
  <list> (intVar wspace)* </list>
  <values> (intVal wspace)* </values>
</instantiation>
```

As an illustration, let us consider the optimization problem from Chapter 1. The optimal solution can be represented by:
Of course, although the syntax is not developed here (because it is immediate), it is possible to deal with other types of variables:

- for symbolic variables, values are represented by symbols as e.g., green, lucie or b,
- for real variables, values are represented by real numbers or intervals as e.g., 7.2 or [3.01, 3.02],
- for set variables, values are represented by set values, as e.g., {2, 3, 7} or {}.

Importantly, when some useless variables are present in an instance, the use is offered three possibilities to deal with them:

- the useless variables (and of course their values) are not listed,
- the useless variables are given the special value '*';
- the useless variables are given any value from their domains.

Let us illustrate this with a small CSP example. The following instance involves an array of 4 variables, but one of them, x[3], is useless.

Example 25

<instance format="XCSP3" type="CSP">
  <variables>
    <array id="x" size="4"> 1..3 </array>
  </variables>
  <constraints>
    <intension> eq(add(x[0],x[1]),x[2]) </intension>
  </constraints>
</instance>

To represent the solution x[0] = 1, x[1] = 1, x[2] = 2, one can choose between the three following representations:

Example 26

<instantiation type="solution">
  <list> x[0], x[1], x[2] </list>
  <values> 1 1 2 </values>
</instantiation>

Example 27

<instantiation type="solution">
  <list> x[] </list>
  <values> 1 1 2 * </values>
</instantiation>
Example 28

```xml
<instantiation type="solution">
  <list> x[] </list>
  <values> 1 1 2 1 </values>
</instantiation>
```

For the last representation, we could have chosen the value 2 or the value 3 for the last variable $x[3]$. Note that the interest of these different forms clearly depends on the context.

**Remark 16** Interestingly, a solution can be perceived as a constraint; see Section 4.1.8.2. The attributes are thus simply ignored.
There are two kinds of elements that can be used for representing objectives. You can use:

- an element `<minimize>`
- or an element `<maximize>`

These elements must be put inside `<objectives>`. The role of the attribute `combination` will be explained at the end of this chapter.

**Syntax 20**

```xml
<objectives [combination="combinationType"]>
  (<minimize.../> | <maximize.../>)+
</objectives>
```

Each element `<minimize>` and `<maximize>` has an optional attribute `id` and an optional attribute `type`, whose value can currently be:

- "expression"
- "sum"
- "product"
- "minimum"
- "maximum"
- "nValues"
- "lex"

### 3.1 Objectives in Functional Forms

The default value for `type` is "expression", meaning that the content of the element `<minimize>` or `<maximize>` is necessarily a numerical functional expression (of course, possibly just a vari-
able) built from operators described in Tables 4.1 and 5.1 for COP instances (that deal with integer values only), and Table 6.1 for NCOP instances (that deal with real values).

An objective in functional form is thus defined by an element <minimize> or <maximize>. We only give the syntax of <minimize> (as the syntax for <maximize> is quite similar) for COP instances. Here, an integer functional expression is referred to as intExpr in the syntax box below; its precise syntax is given in Appendix B.

Syntax 21

```
<minimize [id="identifier"] [type="expression"]>
  intExpr
</minimize>
```

For NCOP instances, just replace above intExpr by realExpr.

An example is given below for an objective obj1 that consists in minimizing the value of the variable z, and an objective obj2 that consists in maximizing the value of the expression $x + 2y$.

Example 29

```
<minimize id="obj1"> z </minimize>
<maximize id="obj2"> add(x, mul(y, 2)) </maximize>
```

This way of representing objectives is generic, but when possible, it is better to use specialized forms in order to simplify the XCSP3 code and also to inform directly solvers of the nature of the objective(s).

### 3.2 Objectives in Specialized Forms

Whatever is the type among "sum", "product", "minimum", "maximum", "nValues" and "lex", two forms are possible. We show this for the element <minimize>, but of course, this is quite similar for the element <maximize>. Here, we give the syntax for COP instances:

Syntax 22

```
<minimize [id="identifier"] type="sum|product|minimum|maximum|nValues|lex">
  [list> (intVar wspace)2+ </list>]
  [coeffs> (intVal wspace)2+ ]</coeffs>]
</minimize>
```

For NCOP instances, you just have to replace (intVar wspace)2+ and (intVal wspace)2+ by (realVar wspace)2+ and (realVal wspace)2+.

There is one possible coefficient per variable. When the element <coeffs> is absent, coefficients are all assumed to be equal to 1, and the opening/closing tags for <list> become optional, which gives:
Syntax 23

```
<minimize [id="identifier"] type="sum|product|minimum|maximum|nValues|lex">
  (intVar wspace)2+ // Simplified Form
</minimize>
```

For the semantics, we consider that $X = \langle x_1, x_2, \ldots, x_k \rangle$ and $C = \langle c_1, c_2, \ldots, c_k \rangle$ denote the lists of variables and coefficients. Also, minimize\textsubscript{lex} denotes minimization over tuples when considering the lexicographic order. Finally, the type is given as third argument of elements \texttt{minimize} below.

**Semantics 1**

- \texttt{minimize(X,C,sum)} : minimize $\sum_{i=1}^{\mid X \mid} c_i \times x_i$
- \texttt{minimize(X,C,product)} : minimize $\prod_{i=1}^{\mid X \mid} c_i \times x_i$
- \texttt{minimize(X,C,minimum)} : minimize $\min_{i=1}^{\mid X \mid} c_i \times x_i$
- \texttt{minimize(X,C,maximum)} : minimize $\max_{i=1}^{\mid X \mid} c_i \times x_i$
- \texttt{minimize(X,C,nValues)} : minimize $\{ c_i \times x_i : 1 \leq i \leq \mid X \mid \}$
- \texttt{minimize(X,C,lex)} : minimize\textsubscript{lex} $\langle c_1 \times x_1, c_2 \times x_2, \ldots, c_k \times x_k \rangle$

The following example shows an objective \texttt{obj1} that consists in minimizing $2x_1 + 4x_2 + 3x_3 + 4x_4 + 8x_5$, and an objective \texttt{obj2} that consists in minimizing the highest value among those taken by variables $y_1, y_2, y_3, y_4$.

**Example 30**

```
<minimize id="obj1" type="sum">
  <list> x1 x2 x3 x4 x5 </list>
  <coeffs> 2 4 1 4 8 </coeffs>
</minimize>
<minimize id="obj2" type="maximum">
  <list> y1 y2 y3 y4 </list>
</minimize>
```

Because the opening and closing tags of \texttt{<list>} are optional here (as there is no element \texttt{<coeffs>}), the objective "obj2" can be simply written:

**Example 31**

```
<minimize id="obj2" type="maximum">
  y1 y2 y3 y4
</minimize>
```

**Remark 17** There is no real interest in introducing coefficients for "product".

### 3.3 Multi-objective Optimization

When dealing with several objectives, it is possible to indicate how these objectives must be combined. The element \texttt{<objectives>} has an optional attribute \texttt{combination}. Current possible values for this attribute are:
• "lexico": the objectives are lexicographically ordered (according to their positions in <objectives>).

• "pareto": no objective is more important than another one.

  Note that this attribute is forbidden when there is only one objective. The default value is "pareto". More values might be introduced in the future.

  For the following example, we try first to minimize the value of $z$, and in case of equality, we try to maximize the value of the expression $x + (y - 10)$.

```xml
Example 32
<objectives combination="lexico">
  <minimize id="obj1"> z </minimize>
  <maximize id="obj2"> add(x, sub(y, 10)) </maximize>
</objectives>
```
Part II

Constraints
In this part of the document, we show how basic forms of constraints are represented in XCSP3. This part contains three chapters, as follows:

- **Constraints**
  - Constraints over Simple Discrete Variables
    - Constraints over Integer Variables
    - Constraints over Symbolic Variables
  - Constraints over Complex Discrete Variables
    - Constraints over Set Variables
    - Constraints over Graph Variables
  - Constraints over Continuous Variables
    - Constraints over Real Variables
    - Constraints over Qualitative Variables
The element `<constraints>` may contain many kinds of elements, as `<constraint.../>` denotes any kind of elementary constraint as shown in Appendix B. For example, this can be `<extension.../>`, `<intension.../>`, `<regular.../>`, and so on. In this chapter, we are interested in constraints over simple discrete variables that are either integer or symbolic variables. First, we focus on (elementary) constraints that involve integer variables. This includes of course constraints involving 0/1 variables, which also serve in our format as Boolean variables. Some of these constraints are also appropriate for symbolic variables, as we shall see at the end of this chapter. Constraints involving set, graph and continuous variables will be introduced in next chapters.

Recall first that there are three main ways of representing constraints:

- by means of a predicate (Boolean expression);
- by listing supports (allowed tuples) or conflicts (forbidden tuples);
- by referring to a known pattern called global constraint.

In this chapter, and the next ones, we describe these different possibilities for XCSP3. Each constraint corresponds to an XML element (except for some special forms allowing us to build groups of constraints) that is put inside the element `<constraints>`. Each constraint has an optional attribute `id`, and contains one or several XML elements that can be seen as the parameters of the constraint. A constraint parameter is an XML element that usually contains a simple term like a numerical value (possibly, an interval), a variable id, or a more complex term like a list of values, a list of (ids of) variables or a list of tuples.

Note that many global constraints have been introduced in CP; see [136, 117, 6]. For a well-detailed documentation, we solicit the reader to consult the global constraint catalog. Below, we introduce constraints per family as shown by Figure 4.1.

For the syntax, we invite the reader to consult Chapter 1 and Appendix B. Recall that BNF
Constraints over Integer Variables

- **Generic Constraints**
  - Constraint intension
  - Constraint extension
  - Constraint smart

- **Language-based Constraints**
  - Constraint regular
  - Constraint grammar
  - Constraint mdd

- **Comparison-based Constraints**
  - Constraints allDifferent, allEqual
  - Constraints allDistant, ordered

- **Counting Constraints**
  - Constraints sum, count
  - Constraints nValues, cardinality
  - Constraints balance, spread, deviation
  - Constraint sumCosts

- **Connection Constraints**
  - Constraints maximum, minimum
  - Constraints element, channel
  - Constraints permutation, precedence

- **Packing and Scheduling Constraints**
  - Constraint stretch
  - Constraint noOverlap
  - Constraint cumulative
  - Constraints binPacking, knapsack, networkFlow

- **Constraints on Graphs**
  - Constraints circuit, nCircuits
  - Constraints path, nPaths
  - Constraints tree, nTrees

- **Elementary Constraints**
  - Constraints clause, instantiation

Figure 4.1: The different types of constraints over integer variables.
non-terminals are written in dark blue italic form, such as for example \textit{intVar} and \textit{intVal} that respectively denote an integer variable and an integer value. In XCSP3, an \textit{intVar} corresponds to the id of a variable declared in \texttt{<variables>} and an \textit{intVal} corresponds to a value in \texttt{N}. Note that when the value of a parameter can be an \textit{intVal} or an \textit{intVar}, we only give the semantics for \textit{intVar}. To simplify the presentation, we omit to specify [id="identifier"] when introducing all following types of constraints.

4.1 Constraints over Integer Variables

4.1.1 Generic Constraints

In this section, we present general ways of representing constraints, namely, intentional and extensional forms of constraints. We also include so-called smart constraints that represent a kind of hybridization between intentional and extensional forms. The elements we introduce are:

1. \textit{intension}  
2. \textit{extension}  
3. \textit{smart}  

4.1.1.1 Constraint intension

Intentional constraints form an important type of constraints. They are defined from Boolean expressions, usually called predicates. For example, the constraint \(x + y = z\) corresponds to an equation that is an expression evaluated to \textit{true} or \textit{false} according to the values assigned to the variables \(x\), \(y\) and \(z\). Predicates are represented under functional form in XCSP3: the function name appears first, followed by the operands between parenthesis (with comma as a separator). The XCSP3 representation of the constraint \(x + y = z\) is \texttt{eq(add(x,y),z)}. Operators on integers (including Booleans since we assume that \texttt{false} = 0 and \texttt{true} = 1) that can be used to build predicates are presented in Table 4.1. When we refer to a Boolean operand or result, we mean either the integer value 0 or the integer value 1. This allows us to combine Boolean expressions with arithmetic operators (for example, addition) without requiring any type conversions. For example, it is valid to write \texttt{eq(add(lt(x,5),lt(y,z)),1)} for stating that exactly one of the Boolean expressions \(x < 5\) and \(y < z\) must be true, although it may be possible (and more relevant) to write it differently.

Remark 18: Everytime we are referring to (the result of) a Boolean expression, do think about either the integer value 0 (false) or the integer value 1 (true).

An intensional constraint is defined by an element \texttt{<intension>}, containing an element \texttt{<function>} that describes the functional representation of the predicate, referred to as \texttt{bool-Expr} in the syntax box below, and whose precise syntax is given in Appendix B.

\begin{verbatim}
Syntax 24
<intension>  
<function> boolExpr </function>  
</intension>
\end{verbatim}

Note that the opening and closing tags for \texttt{<function>} are optional, which gives:
Table 4.1: Operators on integers that can be used to build predicates. As Boolean values false and true are represented by integer values 0 and 1, when an integer (operand/result) is expected, one can provide a Boolean value. In other words, “integer” encompasses “Boolean”. For example, for arithmetic operators, operands can simply be 0/1 (Boolean).

Below, \( P \) denotes a predicate expression with \( r \) formal parameters (not shown here, for...
simplicity), \( X = \langle x_1, x_2, \ldots, x_r \rangle \) a sequence of \( r \) variables, the scope of the constraint, and \( P(x_1, x_2, \ldots, x_r) \) the value (0 or 1) returned by \( P \) for any instantiation of the variables of \( X \).

<table>
<thead>
<tr>
<th>Semantics 2</th>
</tr>
</thead>
</table>
| **intension**(\( X, P \)), with \( X = \langle x_1, x_2, \ldots, x_r \rangle \) and \( P \) a predicate iff 
\( P(x_1, x_2, \ldots, x_r) = 1 \) // recall that 1 stands for true |

For example, for constraints \( c_1 : x + y = z \) and \( c_2 : w \geq z \), we have:

<table>
<thead>
<tr>
<th>Example 33</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;intension id=&quot;c1&quot;&gt; &lt;function&gt; eq(add(x,y),z) &lt;/function&gt; &lt;/intension&gt;</td>
</tr>
<tr>
<td>&lt;intension id=&quot;c2&quot;&gt; &lt;function&gt; ge(w,z) &lt;/function&gt; &lt;/intension&gt;</td>
</tr>
</tbody>
</table>

or, equivalently, in simplified form:

<table>
<thead>
<tr>
<th>Example 34</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;intension id=&quot;c1&quot;&gt; eq(add(x,y),z) &lt;/intension&gt;</td>
</tr>
<tr>
<td>&lt;intension id=&quot;c2&quot;&gt; ge(w,z) &lt;/intension&gt;</td>
</tr>
</tbody>
</table>

Most of the time, intensional constraints correspond to primitive constraints which admit specialized filtering algorithms, such as the binary operators =, \( \neq \), <, ... When parsing, it is rather easy to identify those primitive constraints.

**Remark 19** It is not possible to use compact lists of array variables (see Chapter 2) and "+/-infinity" values in predicates.

### 4.1.1.2 Constraint extension

Extensional constraints, also called table constraints (especially, when constraint arity is large), form another important type of constraints. They are defined by enumerating the list of tuples that are allowed (supports) or forbidden (conflicts). Some algorithms for binary extensional constraints are AC3 [88], AC4 [96], AC6 [15], AC2001 [23], AC3'rm [79] and AC3'di+r+rm [83]. Some algorithms for non-binary extensional constraints are GAC-schema [22], GAC-nextIn [86], GAC-nextDiff [59], GAC-va [82], STR2 [77], AC5TC-Tr [89], STR3 [80] and GAC4r [98]. The state-of-the-art algorithm is CT (Compact-Table), as described in [45, 139]; a filtering algorithm based on similar principles has been independently proposed in [143].

An extensional constraint contains then two elements. The first element is an element <list> that indicates the scope of the constraint (necessarily a list of variable ids). The second element is either an element <supports> or an element <conflicts>, depending on the semantics of the ordinary tuples that are listed in lexicographic order within the element.

For classical non-unary positive table constraints, we have:
Syntax 26
<extension>
  <list> (intVar wspace)2* </list>
  <supports> ("(" intVal "," intVal+ ")")* </supports>
</extension>

Semantics 3
extension\((X, S)\), with \(X = \langle x_1, x_2, \ldots, x_r \rangle\) and \(S\) the set of supports, iff
\(\langle x_1, x_2, \ldots, x_r \rangle \in S\)

Prerequisite: \(\forall \tau \in S, |\tau| = |X| \geq 2\)

For classical non-unary negative table constraints, we have:

Syntax 27
<extension>
  <list> (intVar wspace)2* </list>
  <conflicts> ("(" intVal "," intVal+ ")")* </conflicts>
</extension>

Semantics 4
extension\((X, C)\), with \(X = \langle x_1, x_2, \ldots, x_r \rangle\) and \(C\) the set of conflicts, iff
\(\langle x_1, x_2, \ldots, x_r \rangle \notin C\)

Prerequisite: \(\forall \tau \in C, |\tau| = |X| \geq 2\)

Here is an example with a ternary constraint \(c_1\) defined on scope \(\langle x_1, x_2, x_3 \rangle\) with supports
\(\{\langle 0, 1, 0 \rangle, \langle 1, 0, 0 \rangle, \langle 1, 1, 0 \rangle, \langle 1, 1, 1 \rangle\}\) and a quaternary constraint \(c_2\) defined on scope \(\langle y_1, y_2, y_3, y_4 \rangle\) with conflicts
\(\{\langle 1, 2, 3, 4 \rangle, \langle 3, 1, 3, 4 \rangle\}\).

Example 35
<extension id="c1">
  <list> x1 x2 x3 </list>
  <supports> (0,1,0)(1,0,0)(1,1,0)(1,1,1) </supports>
</extension>

<extension id="c2">
  <list> y1 y2 y3 y4 </list>
  <conflicts> (1,2,3,4)(3,1,3,4) </conflicts>
</extension>

Quite often, when modeling, there are groups of extensional constraints that share the same
relations. It is then interesting to exploit the concept of syntactic group, presented in Section
10.1, or if not appropriate, the attribute as described in Section 10.5.
For unary table constraints, one uses a simplified form: supports and conflicts are indeed defined exactly as the domains of integer variables. It means that we directly put integer values and intervals in increasing order within <supports> and <conflicts>.

For unary positive table constraints, we have:

```
Syntax 28
<extension>
  <list> intVar </list>
  <supports> ((intVal | intIntvl) wspace)* </supports>
</extension>
```

For unary negative table constraints, we have:

```
Syntax 29
<extension>
  <list> intVar </list>
  <conflicts> ((intVal | intIntvl) wspace)* </conflicts>
</extension>
```

The semantics is immediate. As an illustration, the constraint $c_3$ corresponds to $x \in \{1, 2, 4, 8, 9, 10\}$.

```
Example 36
<extension id="c3">
  <list> x </list>
  <supports> 1 2 4 8..10 </supports>
</extension>
```

We have just considered ordinary tables that are tables containing ordinary tuples, i.e., classical sequences of values as in $(1, 2, 0)$. Short tables [97] can additionally contain short tuples, which are tuples involving the special symbol $*$ as in $(0, *, 2)$, and compressed tables [71, 144] can additionally contain compressed tuples, which are tuples involving sets of values as in $(0, \{1, 2\}, 3)$. We now introduce the XCSP3 representation of these extended forms of (non-unary) extensional constraints.

For short table constraints, the syntax is obtained from that of non-unary ordinary table constraints given at the beginning of this section, by replacing $\text{intVal}$ by $\text{intValShort}$ in elements <supports> and <conflicts>; $\text{intValShort}$ is defined by $\text{intVal} | \"*\"$ in Appendix B. The semantics is such that everytime the special symbol $\"*\"$ occurs in a tuple any possible value is accepted for the associated variable(s). For example, the following constraint $c_4$ has two short tuples, $(1, *, 1, 2)$ and $(2, 1, *, *)$. Assuming that the domain of all variables is $\{1, 2\}$, the first short tuple is equivalent to the two ordinary tuples $(1, 1, 1, 2)$ and $(1, 2, 1, 2)$ whereas the second short tuple is equivalent to the four ordinary tuples $(2, 1, 1, 1)$, $(2, 1, 1, 2)$, $(2, 1, 2, 1)$ and $(2, 1, 2, 2)$. 

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For compressed table constraints, the syntax is obtained from that of non-unary ordinary table constraints given at the beginning of this section, by replacing \texttt{intVal} by \texttt{intValCompressed} in elements \texttt{<supports>} and \texttt{<conflicts>}; \texttt{intValCompressed} is defined by \texttt{intVal | "*" | setVal} in Appendix B. The semantics is such that every time a set of values occurs in a tuple any value from this set is accepted for the associated variable(s). The set of ordinary tuples represented by a compressed tuple $\tau$ is then the Cartesian product built from the components of $\tau$. For example, in constraint $c_5$, $(0,\{1,2\},0,\{0,2\})$ is a compressed tuple representing the set of ordinary tuples in $\{0\} \times \{1,2\} \times \{0\} \times \{0,2\}$, which contains $(0,1,0,0)$, $(0,1,0,2)$, $(0,2,0,0)$ and $(0,2,0,2)$.

\begin{example}
<extension id="c4">
  <list> z1 z2 z3 z4 </list>
  <supports> (1,*,1,2)(2,1,*,*) </supports>
</extension>
\end{example}

Remark 20 In any table, ordinary tuples must always be given in lexicographic order, without any repetitions.

4.1.1.3 Constraint \texttt{smart}

A constraint \texttt{smart} \cite{90} is defined from a so-called \texttt{smart} table, authorizing entries to contain simple arithmetic constraints. A smart table is then composed of rows (also called \texttt{smart} tuples), each of them involving restrictions of the form:

1. $x \odot a$
2. $x \in S$,
3. $x \notin S$
4. $x \odot y$
5. $x \odot y + a$

where $x$ and $y$ are variables in the scope of the constraint, $a$ is a value, $S$ is a set of values and $\odot$ is a relational operator in the set $\{<,\leq,\geq,>,=,\neq\}$.

The semantics is the following: an ordinary tuple $\tau$ is allowed by a constraint \texttt{smart} $c$ if there exists at least one row in the table of $c$ such that $\tau$ satisfies all restrictions imposed on the row. Currently, there are two forms of smart constraints that can be enforced to be (generalized) arc-consistent by a dedicated filtering algorithm:

- acyclic smart constraints \cite{90}
- basic smart constraints \cite{138}
It is also possible to transform ordinary tables into smart tables [35]. Smart tables are very useful for modeling, permitting compact and structured representation of constraints. They also facilitate the reformulation of constraints, including well-known global constraints.

As a first illustration, assuming that \( \text{dom}(x_i) = \{1, 2, 3\} \), \( \forall i \in 1..3 \), the following table constraint:

\[
\langle x_1, x_2, x_3 \rangle \in \{ (1, 2, 1), (1, 3, 1), (2, 2, 2), (2, 3, 2), (3, 2, 3), (3, 3, 3) \}
\]

can be represented by the following smart constraint \( c_1 \):

\[
\begin{array}{ccc}
x_1 & x_2 & x_3 \\
= x_3 & \geq 2 & *
\end{array}
\]

As a second illustration, let us consider a car configuration problem. We assume that the cars to be configured have 2 colors (one for the body, \( \text{colB} \), and the other for the roof, \( \text{colR} \)), a model number \( \text{mod} \), an option pack \( \text{pck} \), and an onboard computer \( \text{cmp} \). A configuration rule states that, for a particular model number \( a = 2 \) and some fancy body color set \( S = \{3, 5\} \), an option pack less than a certain pack \( b = 4 \) implies that the onboard computer cannot be the most powerful one, \( c = 3 \), and that the roof color has to be the same as the body color. We obtain:

\[
(\text{mod} = a \land \text{colB} \in S \land \text{pck} < b) \Rightarrow (\text{cmp} \neq c \land \text{colR} = \text{colB})
\]

that can be represented under a disjunctive form:

\[
\text{mod} \neq a \lor \text{colB} \notin S \lor \text{pck} \geq b \lor (\text{cmp} \neq c \land \text{colR} = \text{colB})
\]

which gives the following smart constraint \( c_2 \):

\[
\begin{array}{cccc}
\text{mod} & \text{colB} & \text{colR} & \text{pck} & \text{cmp} \\
\neq 2 & * & * & * & * \\
* & \notin \{3, 5\} & * & * & * \\
* & * & * & \geq 4 & * \\
* & * & = \text{colB} & * & \neq 3
\end{array}
\]

Finally, as an illustration of a global constraint that can be reformulated as a smart table constraint, we have:

\[
\text{maximum}(\langle x_1, x_2, x_3, x_4 \rangle) = y
\]

which gives the following smart constraint \( c_3 \):

\[
\begin{array}{cccc}
x_1 & x_2 & x_3 & x_4 & y \\
* & \leq x_1 & \leq x_1 & \leq x_1 & = x_1 \\
\leq x_2 & * & \leq x_2 & \leq x_2 & = x_2 \\
\leq x_3 & \leq x_2 & * & \leq x_2 & = x_3 \\
\leq x_4 & \leq x_4 & \leq x_4 & * & = x_4
\end{array}
\]

A smart constraint contains a first element \(<\text{list}>\) that indicates the scope of the constraint (necessarily a list of variable ids). Then, it contains a non-empty sequence of elements \(<\text{row}>\), each of them being composed of a prefix and a suffix separated by the symbol ".". The prefix is either "*" (denoting a short tuple only containing "*") or an ordinary or short tuple. The suffix is a (possibly empty) sequence of restrictions as defined at the beginning of this section. With \textit{smartExpr}, defined in Appendix B, and \textit{intValShort} defined by \textit{intVal} \mid "*", the syntax is:
The XCSP3 representation of the smart constraints introduced earlier in this section is:

Example 39

```xml
<smart id="c1">
  <list> x1 x2 x3 </list>
  <row> * : eq(x1,x3) ge(x2,2) </row>
</smart>

<smart id="c2">
  <list> mod colB colR pck cmp </list>
  <row> * : ne(mod,2) </row>
  <row> * : in(colB,set(3,5)) </row>
  <row> * : ge(pck,4) </row>
  <row> * : eq(colR,colB) ne(cmp,3) </row>
</smart>

<smart id="c3">
  <list> x1 x2 x3 x4 y </list>
  <row> * : le(x2,x1) le(x3,x1) le(x4,x1) eq(y,x1) </row>
  <row> * : le(x1,x2) le(x3,x2) le(x4,x2) eq(y,x2) </row>
  <row> * : le(x1,x3) le(x2,x3) le(x4,x3) eq(y,x3) </row>
  <row> * : le(x1,x4) le(x2,x4) le(x3,x4) eq(y,x4) </row>
</smart>
```

Of course, in some occasions, it may be useful to use a tuple instead of "*" as a prefix of a row. For example, the following smart constraint \( c_4 \):

Example 40

```xml
<smart id="c4">
  <list> z1 z2 z3 z4 </list>
  <row> (2,* ,4,* ) : eq(z2,z4) </row>
  <row> (3,1,* ,2) : le(z3,5) </row>
</smart>
```

is a representation that is more compact and readable than the following equivalent representation:
4.1.2 Constraints defined from Languages

In this section, we present constraints that are defined from advanced data structures such as automatas, grammars and diagrams, which exhibit languages. More specifically, we introduce:

1. regular
2. grammar
3. mdd

4.1.2.1 Constraint regular

The constraint regular [33, 100] ensures that the sequence of values assigned to the variables it involves forms a word that can be recognized by a deterministic (or non-deterministic) finite automaton. The scope of the constraint is given by the element <list>, and three elements, <transitions>, <start> and <final>, are introduced for representing respectively the transitions, the start state and the final (accept) states of the automaton. Note that the set of states and the alphabet can be inferred from <transitions>. When the automaton is non-deterministic, we can find two transitions \((q_i, a, q_j)\) and \((q_i, a, q_k)\) such that \(q_j \neq q_k\). In what follows, state stands for any identifier and states for a sequence of identifiers (whitespace as separator).

**Syntax 31**

```xml
<regular>
  <list> (intVar wspace)+ </list>
  <transitions> ("(" state "," intVal "," state ")")+ </transitions>
  <start> state </start>
  <final> (state wspace)+ </final>
</regular>
```

Below, \(L(A)\) denotes the language recognized by a deterministic (or non-deterministic) finite automaton \(A\).

**Semantics 5**

\[ \text{regular}(X, A), \text{ with } X = \langle x_1, x_2, \ldots, x_r \rangle \text{ and } A \text{ a finite automaton, iff } x_1, x_2, \ldots, x_r \in L(A) \]
As an example, the constraint defined on scope \( \langle x_1, x_2, \ldots, x_7 \rangle \) from the simple automation depicted above is:

```xml
<regular>
<list> x1 x2 x3 x4 x5 x6 x7 </list>
<transitions>
(a,0,a)(a,1,b)(b,1,c)(c,0,d)(d,0,d)(d,1,e)(e,0,e)
</transitions>
<start> a </start>
<final> e </final>
</regular>
```

### 4.1.2.2 Constraint grammar

The constraint grammar [126, 108, 68, 109, 69] ensures that the sequence of values assigned to the variables it involves belongs to the language defined by a formal grammar. The scope of the constraint is given by the element `<list>`, and three elements, `<terminal>`, `<rules>` and `<start>`, are introduced for representing respectively the terminal symbols (in our case, integer values), the production rules and the start non-terminal symbol of the grammar. Each rule is composed of a “head” word (on the left) containing an arbitrary number of symbols (provided that at least one of them is a non-terminal symbol) and a “body” word (on the right) containing an arbitrary number of symbols (possibly, none). It is important to note that both the head and body words are represented by a sequence of symbols, using whitespace as a separator between each pair of consecutive symbols. This avoids ambiguity, and besides, this allows us to infer the set of non-terminal symbols from `<terminal>` and `<rules>`. In what follows, `symbol` stands for any identifier.

```xml
<grammar>
<list> (intVar wspace)* </list>
<terminal> (intVal wspace)* </terminal>
<rules>
("(" (intVal | symbol) (wspace (intVal | symbol))*) ",
 [ (intVal | symbol) (wspace (intVal | symbol))]* ")")+
</rules>
<start> symbol </start>
</grammar>
```

Below, \( L(G) \) denotes the language recognized by a formal grammar \( G \).

```xml
Semantics 6
grammar(\( (X,G) \), with \( X = (x_1, x_2, \ldots, x_r) \) and \( G \) a grammar, iff \( x_1, x_2, \ldots, x_r \in L(G) \))
```

57
As an example, let us consider the grammar defined from the set of non-terminal symbols \( N = \{ A, S \} \) (\( S \) being the start symbol), the set of terminal symbols \( \Sigma = \{ 0, 1, 2 \} \), and the following rules (with \( \epsilon \) denoting the empty string):

- \( S \rightarrow 0S \)
- \( S \rightarrow 1S \)
- \( A \rightarrow \epsilon \)
- \( A \rightarrow 2A \)

This grammar describes the same language as the regular expression \( 0^*12^* \). Assuming that \( x_1, x_2 \) and \( x_3 \) are three integer variables with domain \( \{ 0, 1, 2 \} \), we could build a constraint grammar as follows:

Example 43

```
<grammar>
  <list> x1 x2 x3 </list>
  <terminal> 0 1 2 </terminal>
  <rules> (S,0 S)(S,1 S)(A,)(A,2 A) </rules>
  <start> S </start>
</grammar>
```

4.1.2.3 Constraint mdd

The constraint mdd [36, 37, 38, 98] ensures that the sequence of values assigned to the variables it involves follows a path going from the root of the described MDD (Multi-valued Decision Diagram) to the unique terminal node. Because the graph is directed, acyclic, with only one root node and only one terminal node, we just need to introduce `<transitions>`.

Syntax 33

```
<mdd>
  <list> (intVar wspace)+ </list>
  <transitions> ("(" state "," intVal "," state ")")+ </transitions>
</mdd>
```

Below, \( L(M) \) denotes the language recognized by a MDD \( M \).
As an example, the constraint of scope $\langle x_1, x_2, x_3 \rangle$ is defined from the simple MDD depicted above (with root node $r$ and terminal node $t$) as:

```
<example 44>
  <mdd>
    <list> x1 x2 x3 </list>
    <transitions>
      (r,0,n1)(r,1,n2)(r,2,n3)
      (n1,2,n4)(n2,2,n4)(n3,0,n5)
      (n4,0,t)(n5,0,t)
    </transitions>
  </mdd>
</example 44>
```

### 4.1.3 Comparison-based Constraints

In this section, we present constraints that are based on comparisons between pairs of variables. More specifically, we introduce:

1. allDifferent (capturing allDifferentExcept)
2. allEqual
3. allDistant
4. ordered

#### 4.1.3.1 Constraint allDifferent

The constraint allDifferent, see [112, 135, 60], ensures that the variables in $\langle\text{list}\rangle$ must all take different values. A variant, called allDifferentExcept in the literature [6, 41], enforces variables to take distinct values, except those that are assigned to some specified values (often, the single value 0). This is the reason why we introduce an optional element $\langle\text{except}\rangle$. 

59
Note that the opening and closing tags of `<list>` are optional if `<list>` is the unique parameter of the constraint, which gives:

**Syntax 35**

```
<allDifferent> (intVar wspace)+ </allDifferent>  // Simplified Form
```

**Semantics 8**

allDifferent\((X,E)\), with \(X = \langle x_1, x_2, \ldots \rangle\), iff

\[
\forall (i,j) : 1 \leq i < j \leq |X|, x_i \neq x_j \lor x_i \in E \lor x_j \in E
\]

allDifferent\((X)\) iff allDifferent\((X,\emptyset)\)

For example, below, the constraint \(c_1\) forces all variables \(x_1, x_2, x_3, x_4, x_5\) to take different values, and the constraint \(c_2\) forces each of the variables of the 1-dimensional array \(y\) to take either the value 0 or a value different from the other variables.

**Example 45**

```
<allDifferent id="c1">
x1 x2 x3 x4 x5
</allDifferent>
<allDifferent id="c2">
<list> y[] </list>
<except> 0 </except>
</allDifferent>
```

**Remark 21** Possible restricted forms of allDifferent are possible by forcing variables to be, for example, symmetric and/or irreflexive. For more details, see Section 7.3.

Although that, for simplicity, the syntax and semantics in this document are given for rather strict forms, one should be aware that the format can accept very easily more sophisticated forms. This is related to the concept of view, as discussed in Section 10.4. As an illustration, it is possible to use a variant of allDifferent where integer variables are replaced by integer expressions, as in:

**Example 46**

```
<allDifferent>
  add(x1,1) add(x2,2) add(x3,3)
</allDifferent>
```
4.1.3.2 Constraint allEqual

The constraint allEqual ensures that all involved variables take the same value.

Syntax 36

```xml
<allEqual>
  <list> (intVar wspace)2+ </list>
</allEqual>
```

Note that the opening and closing tags of <list> are optional, which gives:

Syntax 37

```xml
<allEqual> (intVar wspace)2+ </allEqual> // Simplified Form
```

Semantics 9

\[ \text{allEqual}(X), \text{with } X = \langle x_1, x_2, \ldots \rangle, \text{iff} \]
\[ \forall (i, j) : 1 \leq i < j \leq |X|, x_i = x_j \]

Example 47

```xml
<allEqual>
  x1 x2 x3 x4 x5
</allEqual>
```

4.1.3.3 Constraint allDistant

The constraint allDistant ensures that the distance between each pair of variables of <list> is subject to a numerical condition. In [114, 106], this constraint is studied with respect to the operator \( \geq \), and called interDistance.

Syntax 38

```xml
<allDistant>
  <list> (intVar wspace)2+ </list>
  <condition> "(" operator "," operand ")" </condition>
</allDistant>
```

Semantics 10

\[ \text{allDistant}(X, (\circ, k)), \text{with } X = \langle x_1, x_2, \ldots \rangle, \text{iff} \]
\[ \forall (i, j) : 1 \leq i < j \leq |X|, |x_i - x_j| \circ v \]

Below, the constraint \( c_1 \) corresponds to \( |x_1 - x_2| \geq 2 \land |x_1 - x_3| \geq 2 \land |x_2 - x_3| \geq 2 \), whereas the constraint \( c_2 \) guarantees that \( \forall (i, j) : 1 \leq i < j \leq 4 \), we have \( 2 \leq |y_i - y_j| \leq 4 \)
4.1.3.4 Constraint ordered

The constraint `ordered` ensures that the variables of `<list>` are ordered in sequence according to a relational operator specified in `<operator>`, whose value must be in `{<, ≤, ≥, >}`. The optional element `<lengths>` indicates the minimum distances between any two successive variables of `<list>`. 

**Syntax 39**

```xml
<ordered>
  <list> (intVar wspace)2+ </list>
  [<lengths> (intVal wspace)2+ | (intVar wspace)2+ </lengths>]  
  <operator> "lt" | "le" | "ge" | "gt" </operator>
</ordered>
```

**Semantics 11**

ordered\((X, L, \circ), \text{ with } X = \langle x_1, x_2, \ldots \rangle, L = \langle l_1, l_2, \ldots \rangle \text{ and } \circ \in \{<, \leq, \geq, >\}, \text{ iff}
\forall i: 1 \leq i < |X|, x_i + l_i \circ x_{i+1}

ordered\((X, \circ), \text{ with } X = \langle x_1, x_2, \ldots \rangle \text{ and } \circ \in \{<, \leq, \geq, >\}, \text{ iff}
\forall i: 1 \leq i < |X|, x_i \circ x_{i+1}

Prerequisite: \(|X| = |L| + 1\)

Below, the constraint \(c_1\) is equivalent to \(x_1 < x_2 < x_3 < x_4\), and the constraint \(c_2\) is equivalent to \(y_1 + 5 \geq y_2 \land y_2 + 3 \geq y_3\). 

**Example 49**

```xml
<ordered id="c1">
  <list> x1 x2 x3 x4 </list>
  <operator> lt </operator>
</ordered>

<ordered id="c2">
  <list> y1 y2 y3 </list>
  <lengths> 5 3 </lengths>
  <operator> ge </operator>
</ordered>
```
The constraints from the Global Constraint Catalog that are captured by \textit{ordered} are:

- increasing, strictly increasing
- decreasing, strictly decreasing

As a matter of fact, if the element \texttt{<lengths>} is absent, one can define a constraint \texttt{ordered} without using the element \texttt{<operator>}: it suffices to add a special attribute \texttt{case} to \texttt{<ordered>}. This way, the opening and closing tags of \texttt{<list>} become optional, making the XCSP3 representation more compact.

\begin{verbatim}
Syntax 40
| <ordered case="orderedType"> (intVar wspace)2+ </ordered> // Simplified Form
\end{verbatim}

The possible values for the attribute \texttt{case} are "increasing", "strictlyIncreasing", "decreasing" and "strictlyDecreasing". The constraint $c_1$, introduced above, can then be written:

\begin{verbatim}
Example 50
| <ordered id="c1" case="strictlyIncreasing"> x1 x2 x3 x4 </ordered>
\end{verbatim}

Remark 22 \textit{In the future, the format might be extended to include an element specifying the order (preferences between values) to follow.}

4.1.4 Counting and Summing Constraints

In this section, we present constraints that are based on counting the number of times variables or values satisfy a certain condition, and also those that are based on summations. More specifically, we introduce:

1. \texttt{sum} (sometimes called \texttt{linear} in the literature)
2. \texttt{count} (capturing \texttt{among}, \texttt{atLeast}, \texttt{atMost} and \texttt{exactly})
3. \texttt{nValues} (capturing \texttt{nValuesExcept})
4. \texttt{cardinality}
5. \texttt{balance}
6. \texttt{spread}
7. \texttt{deviation}
8. \texttt{sumCosts}

4.1.4.1 Constraint \texttt{sum}

The constraint \texttt{sum} is one of the most important constraint. When the optional element \texttt{<coeffs>} is missing, it is assumed that all coefficients are equal to 1. The constraint is subject to a numerical condition.
Although in practice, coefficients are most the time integer values, we introduce the semantics with variable coefficients.

The following constraint $c_1$ states that the values taken by variables $x_1, x_2, x_3$ and $y$ must respect the linear function $x_1 \times 1 + x_2 \times 2 + x_3 \times 3 > y$.

A form of \texttt{sum}, sometimes called \texttt{subset-sum} or \texttt{knapsack} \cite{129, 101} involves the operator “in”, and ensures that the obtained sum belongs (or not) to a specified interval. The following constraint $c_2$ states that the values taken by variables $y_1, y_2, y_3, y_4$ must respect $2 \leq y_1 \times 4 + y_2 \times 2 + y_3 \times 3 + y_4 \times 1 \leq 5$.

As for \texttt{allDifferent}, it is often useful to accept forms of \texttt{sum} where integer variables are replaced by integer expressions. The tools, modeling API and parsers, that we develop together with XCSP3 can deal with some of these forms (see also Section 10.4): At the time of writing, it concerns \texttt{allDifferent} and \texttt{sum}. As an illustration for \texttt{sum}, the constraint $(x_1 = 1) + (x_2 > 2) + (x_3 = 1) \leq 2$ can be written as:
4.1.4.2 Constraint count

The constraint count, introduced in CHIP [8] and Sicstus [34], ensures that the number of variables in <list> which are assigned a value in <values> respects a numerical condition. It is also present in Gecode with the same name and in the Global Constraint Catalog where it is called counts. This constraint captures known constraints atLeast, atMost, exactly and among.

Syntax 42

To simplify, we assume for the semantics that V is a set of integer values.

Semantics 13

Below, \(c_1\) enforces that the number of variables in \(\{v_1, v_2, v_3, v_4\}\) that take the value of variable \(v\) must be different from the value of \(k_1\). Constraints \(c_2, c_3, c_4\) and \(c_5\) illustrate how to represent atLeast, atMost, exactly and among.

\(c_2\) represents among\((k_2, \{w_1, w_2, w_3, w_4\}, \{1, 5, 8\})\);

\(c_3\) represents atLeast\((k_3, \{x_1, x_2, x_3, x_4, x_5\}, 1)\);

\(c_4\) represents atMost\((2, \{y_1, y_2, y_3, y_4\}, 0)\);

\(c_5\) represents exactly\((k_5, \{z_1, z_2, z_3\}, z)\).

Example 54

\(<\text{count } id="c1"/>\)
\(<\text{list}> v1 v2 v3 v4 </\text{list}>\)
\(<\text{values}> v </\text{values}>\)
\(<\text{condition}> (\text{ne}, k1) </\text{condition}>\)
\(</\text{count}>\)

\(<\text{count } id="c2"/> \ // \ among\)
\(<\text{list}> w1 w2 w3 w4 </\text{list}>\)
\(<\text{values}> 1 5 8 </\text{values}>\)
4.1.4.3 Constraint nValues

The constraint \texttt{nValues} \cite{17}, ensures that the number of distinct values taken by variables in \texttt{<list>} respects a numerical condition. A variant, called \texttt{nValuesExcept} \cite{17} discards some specified values (often, the single value 0). This is the reason why we introduce an optional element \texttt{<except>}

\begin{verbatim}
<condition> (eq,k2) </condition>
</count>
<count id="c3" > // atLeast
  <list> x1 x2 x3 x4 x5 </list>
  <values> 1 </values>
  <condition> (ge,k3) </condition>
</count>
<count id="c4" > // atMost
  <list> y1 y2 y3 y4 </list>
  <values> 0 </values>
  <condition> (le,2) </condition>
</count>
<count id="c5"> // exactly
  <list> z1 z3 z3 </list>
  <values> z </values>
  <condition> (eq,k5) </condition>
</count>
\end{verbatim}

\begin{quote}
4.1.4.3 Constraint nValues

The constraint \texttt{nValues} \cite{17}, ensures that the number of distinct values taken by variables in \texttt{<list>} respects a numerical condition. A variant, called \texttt{nValuesExcept} \cite{17} discards some specified values (often, the single value 0). This is the reason why we introduce an optional element \texttt{<except>}

\begin{verbatim}
<condition> (eq,k2) </condition>
</count>
<count id="c3" > // atLeast
  <list> x1 x2 x3 x4 x5 </list>
  <values> 1 </values>
  <condition> (ge,k3) </condition>
</count>
<count id="c4" > // atMost
  <list> y1 y2 y3 y4 </list>
  <values> 0 </values>
  <condition> (le,2) </condition>
</count>
<count id="c5"> // exactly
  <list> z1 z3 z3 </list>
  <values> z </values>
  <condition> (eq,k5) </condition>
</count>
\end{verbatim}

\begin{quote}
\textbf{Syntax 43}

\begin{verbatim}
<nValues>
  <list> (intVar wspace)2+ </list>
  [<except> (intVal wspace)+ </except>]
  <condition> "( operator "," operand ")" </condition>
</nValues>
\end{verbatim}

\begin{quote}
\textbf{Semantics 14}

\begin{verbatim}
nValues(X,E,(\odot,k)), with X = \langle x_1,x_2,... \rangle, iff 
|\{x_i : 1 \leq i \leq |X| \} \setminus E| \odot k 
nValues(X,(\odot,k)) iff nValues(X,\emptyset,(\odot,k))
\end{verbatim}

\begin{quote}
\textbf{Remark 23} This element captures \texttt{atLeastNValues} and \texttt{atMostNValues}, since it is possible to specify the relational operator in \texttt{<condition>}

In the following example, the constraint \texttt{c1} states that there must be exactly three distinct values taken by variables \texttt{x1},...,\texttt{x4}, whereas \texttt{c2} states at most \texttt{w} distinct values must be taken by variables \texttt{y1},...,\texttt{y5}. The constraint \texttt{c3} ensures that two different values are taken by variables \texttt{z1},...,\texttt{z4}, considering that the value 0 must be ignored.
\end{quote}
\end{quote}
\end{quote}

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Example 55

```xml
<nValues id="c1">
  <list> x1 x2 x3 x4 </list>
  <condition> (eq, 3) </condition>
</nValues>
<nValues id="c2">
  <list> y1 y2 y3 y4 y5 </list>
  <condition> (le, w) </condition>
</nValues>
<nValues id="c3">
  <list> z1 z2 z3 z4 </list>
  <except> 0 </except>
  <condition> (eq, 2) </condition>
</nValues>
```

Remark 24  The constraint increasing NValues can be built by adding restriction increasing to <list>. For more details about restricted constraints, see section 7.3.

4.1.4.4 Constraint cardinality

The constraint cardinality, also called globalCardinality or gcc in the literature, see [113, 67], ensures that the number of occurrences of each value in <values>, taken by variables of <list>, is related to a corresponding element (value, variable or interval) in <occurs>. The element <values> has an optional attribute closed ("false", by default): when closed="true", this means that all variables of <list> must be assigned a value from <values>.

Syntax 44

```xml
<cardinality>
  <list> (intVar wspace)2+ </list>
  <values [closed="boolean!"]> (intVal wspace)+ | (intVar wspace)+ </values>
  <occurs> (intVal wspace)+ | (intVar wspace)+ | (intIntvl wspace)+ </occurs>
</cardinality>
```

For simplicity, for the semantics, we assume that V only contains values and O only contains variables. Note that V cl means that closed="true".

Semantics 15

```latex
\text{cardinality}(X,V,O), \text{ with } X = \langle x_1, x_2, \ldots \rangle, \ V = \langle v_1, v_2, \ldots \rangle, \ O = \langle o_1, o_2, \ldots \rangle,
\text{ if } \forall j: 1 \leq j \leq |V|, |\{i: 1 \leq i \leq |X| \land x_i = v_j\}| = o_j
\text{ cardinality}(X,V,\text{cl}) \iff \text{ cardinality}(X,V,O) \land \forall i: 1 \leq i \leq |X|, x_i \in V
\text{ Prerequisite } : |X| \geq 2 \land |V| = |O| \geq 1
```

In the following example, c1 states that the value 2 must be assigned to 0 or 1 variable (from \{x1, x2, x3, x4\}), the value 5 must be assigned to at least 1 and at most 3 variables, and the value 10 must be assigned to at least 2 and at most 3 variables. Note that it is possible for a variable of c1 to be assigned a value not present in \{2, 5, 10\} since by default we have

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closed="false". For \(c_2\), the number of variables from \(\{y_1, y_2, y_3, y_4, y_5\}\) that take value 0 must be equal to \(z_0\), and so on. Note that it is not possible for a variable \(y_i\) to be assigned a value not present in \(\{0, 1, 2, 3\}\) since closed="true".

Example 56

```xml
<cardinality id="c1">
  <list> x1 x2 x3 x4 </list>
  <values> 2 5 10 </values>
  <occurs> 0..1 1..3 2..3 </occurs>
</cardinality>
<cardinality id="c2">
  <list> y1 y2 y3 y4 y5 </list>
  <values closed="true"> 0 1 2 3 </values>
  <occurs> z0 z1 z2 z3 </occurs>
</cardinality>
```

The form of the constraint obtained by only considering variables in all contained elements is called distribute in MiniZinc. In that case, for the semantics, we must additionally guarantee:

\[ \forall(i, j) : 1 \leq i < j \leq |V|, v_i \neq v_j. \]

Example 57

```xml
<cardinality id="c3">
  <list> w1 w2 w3 w4 </list>
  <values> v1 v2 </values>
  <occurs> n1 n2 </occurs>
</cardinality>
```

The constraint cardinalityWithCosts [116] will be discussed in Section 8.5.3.

**Remark 25** The constraint increasing_global_cardinality defined in the Global Constraint Catalog can be built by adding restriction increasing to \(<list>\). For more details about restricted constraints, see section 7.3.

4.1.4.5 Constraint balance

The constraint balance [6, 21] ensures that the difference between the maximum number of occurrences and the minimum number of occurrences among the values assigned to the variables in \(<list>\) respects a numerical condition. If the optional element \(<values>\) is present, then all variables must be assigned to a value from this set; see balance\(^*\) in [21].

Syntax 45

```xml
<balance>
  <list> (intVar wspace)2+ </list>
  <values> (intVal wspace)+ </values>
  <condition> "(" operator "," operand ")" </condition>
</balance>
```
balance($X$,$(\circ,k)$), with $X = (x_1, x_2, \ldots)$, iff
\[
\max_{v \in V} \{|i : 1 \leq i \leq |X| \land x_i = v\} - \min_{v \in V} \{|i : 1 \leq i \leq |X| \land x_i = v\} \circ k
\]
with $V = \{x_i : 1 \leq i \leq |X|\}$

$balance(X, V, (\circ, k))$ iff $balance(X, (\circ, k)) \land \forall x \in X, x \in V$

Example 58

<balance id="c1">
  <list> x1 x2 x3 x4 </list>
  <condition> (eq, k) </condition>
</balance>

<balance id="c2">
  <list> y1 y2 y3 y4 y5 </list>
  <values> 0 1 2 </values>
  <condition> (lt, 2) </condition>
</balance>

4.1.4.6 Constraint spread

The constraint `spread` [102, 123, 122] ensures that the variance of values taken by variables of `<list>` respects a numerical condition. The element `<total>` is optional; if present, its value must be equal to the sum of values of variables in `<list>`.

Syntax 46

<spread>
  <list> (intVar wspace)2+ </list>
  [<total> intVar </total>]
  <condition> "(" operator "," operand ")" </condition>
</spread>

Semantics 17

spread($X$, $s$, $(\circ,k)$), with $X = (x_1, x_2, \ldots)$, iff
\[
s = \sum_{i=1}^{r} x_i
\]
\[
(r \sum_{i=1}^{r} x_i^2 - s^2) \circ k
\]

4.1.4.7 Constraint deviation

The constraint `deviation` [124, 122] ensures that the deviation of values taken by variables of `<list>` respects a numerical condition. The element `<total>` is optional; if present, its value must be equal to the sum of values of variables in `<list>`.

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Syntax 47

<deviation>
  <list> (intVar wspace)2+ </list>
  [total intVar </total>]
  <condition> "(" operator "," operand ")" </condition>
</deviation>

Semantics 18

deviation(X, s, (⊙, v)), with X = \{x_1, x_2, ..., x_r\}, iff

\[ s = \sum_{i=1}^{r} x_i \]
\[ \sum_{i=1}^{r} |x_i - s| \odot v \]

4.1.4.8 Constraint sumCosts

The constraint sumCosts ensures that the sum of integer costs for values taken by variables in <list> respects a numerical condition. This constraint is particularly useful, when combined with other constraints, to express preferences (as shown in Section 8.5).

A first form of sumCosts allows us to define a cost matrix, where an integer cost is associated with each variable-value pair that we call literal. The matrix contains tuples of integer values; there is one tuple per variable, containing as many costs as values present in the domain of the variable.

Syntax 48

<sumCosts>
  <list> (intVar wspace)2+ </list>
  <costMatrix> ("\" intVal ( ",\" intVal)\")2+ </costMatrix>
  <condition> "\" operator \",\" operand \")\" </condition>
</sumCosts>

For the semantics, we assume that M is a matrix that gives the cost M[x_i][a_i] for any value a_i = x_i assigned to the ith variable x_i of <list>.

Semantics 19

sumCosts(X, M, (⊙, k)), with X = \{x_1, x_2, ..., x_r\}, iff

\[ \sum_{1 \leq i \leq |X|} M[x_i][x_i] \odot k \]

The following example shows a constraint c_1 that involves 3 variables y_1, y_2, y_3 in its main list, with four values in each variable domain (let us say \{0, 1, 2, 3\}). The assignment costs are as follows: 10 for (y_1, 0), 0 for (y_1, 1), 5 for (y_1, 2), 0 for (y_1, 3), 0 for (y_2, 0), 5 for (y_2, 1), and so on. For c_1, the sum of assignment costs must be less than or equal to 12.
A second form of \texttt{sumCosts} allows us to define the cost matrix under a dual point of view. We replace the element \texttt{<costMatrix>} by a sequence of elements \texttt{<literals>}, where each such element has a required attribute \texttt{cost} that gives the common cost of all literals (variable-value pairs) contained inside the element. It is possible to use the optional attribute \texttt{defaultCost} for \texttt{<sumCosts>} to specify the cost of all implicit literals (i.e., those not explicitly listed).

For the semantics, we assume, for any pair \((x_i, a_i)\), that \(\text{cost}^A(x_i, a_i)\) denotes the cost \(\text{cost}(A_j)\) of the set \(A_j\) such that \((x_i, a_i) \in A_j\).

When a value \(a\) always admits the same cost, whatever the variable \(x_i\) is, we can simplify \((x_1, a), (x_2, a), \ldots\) by \((*, a)\).

We give an illustration. The constraint \(c_{1b}\) is equivalent to constraint \(c_1\) introduced above. The constraint \(c_2\) involves 4 variables \(w_1, w_2, w_3, w_4\) in its main list, with 3 values in each domain (let us say \(\{0,1,2\}\)). The assignment costs are as follows: 10 for \((w_1, 0), (w_2, 0), (w_3, 0), (w_4, 0)\) and \((w_3, 1)\), and 0 for the other literals (variable-value pairs).
Example 60

```xml
<sumCosts id="c1b" defaultCost="0">
  <list>
    y1 y2 y3
  </list>
  <literals cost="10"> (y1,0) (y3,1) </literals>
  <literals cost="5"> (y1,2) (y2,1) (y3,0) </literals>
  <condition> (le,12) </condition>
</sumCosts>
<sumCosts id="c2" defaultCost="0">
  <list>
    w1 w2 w3 w4
  </list>
  <literals cost="10"> (*,0) (w3,1) </literals>
  <condition> (eq,z) </condition>
</sumCosts>
```

A related form of \texttt{sumCosts} is defined by using a sequence of elements \texttt{<values>}, where each such element has a required attribute \texttt{cost} that gives the common cost of all values contained inside the element, i.e., whatever the variables are. The optional attribute \texttt{defaultCost} for \texttt{<sumCosts>} is useful for specifying the cost of all implicit values (i.e., those not explicitly listed).

Syntax 50

```xml
<sumCosts [defaultCost="integer"]>
  <list>
    intVar wspace
  </list>
  (<values cost="integer"> intVal wspace </values>)+
  <condition> "( operator "," operand ")" </condition>
</sumCosts>
```

For the semantics, we assume, for any value \( a_i \) in the domain of a variable of \( X \), that \( \text{cost}^V(a_i) \) denotes the cost \( \text{cost}(V_j) \) of the set \( V_j \) such that \( a_i \in V_j \).

Semantics 21

\[ \sum \{ \text{cost}^V(x_i) : 1 \leq i \leq |X| \} \odot k \]

Prerequisite:

\[ \forall(i,j) : 1 \leq i < j \leq |V|, V_i \cap V_j = \emptyset \]
\[ \forall a \in \bigcup_{x \in \text{dom}(x)} V_i, \exists V_i \in V : a \in V_i \]

The constraint \( c_3 \) involves 5 variables \( v_1, v_2, v_3, v_4, v_5 \) in its main list, with 4 values in each domain (let us say \( \{1, 2, 3, 4\} \)). The assignment costs are as follows: 10 for \( (v_1,2) \), \( (v_2,2) \), ..., \( (v_1,4) \), \( (v_2,4) \), ..., and 0 for the other literals. The constraint \( c_3 \) forces \( z \) to be equal to the sum of the assignment costs.
Example 61

```xml
<sumCosts id="c3" defaultCost="0">
  <list> v1 v2 v3 v4 v5 </list>
  <values cost="10"> 2 4 </values>
  <condition> (eq,z) </condition>
</sumCosts>
```

Finally, by replacing the element `<list>` by an element `<set>`, we can represent the constraint `sum_of_weights_of_distinct_values` [7], where any cost is defined per value and can be taken into account only once.

Syntax 51

```xml
<sumCosts [defaultCost="integer"]>
  <set> (intVar wspace)</set>
  <values cost="integer"> (intVal wspace)</values>
  <condition> "(" operator "," operand ")" </condition>
</sumCosts>
```

For the semantics of this constraint variant, that we refer to as `sumCosts_set`, we assume, for any value `a_i` in the domain of a variable of `X`, that `cost^V(a_i)` denotes the cost `cost(V_j)` of the set `V_j` such that `a_i ∈ V_j`.

Semantics 22

\[
\text{sumCosts}_\text{set} (X, \mathcal{V}, (\circ, k)), \text{ with } X = (x_1, x_2, \ldots), \text{ and } \mathcal{V} = (V_1, V_2, \ldots) \text{ where } V_j \text{ is a set of values of cost } cost(V_j), \text{ iff }
\]
\[
\sum \{cost^V(a) : a ∈ \bigcup_{x ∈ X} dom(x) \land \exists i : 1 ≤ i ≤ |X| \land x_i = a\} \circ k
\]

Prerequisite:

\[
\forall (i, j) : 1 ≤ i < j ≤ |\mathcal{V}|, V_i \cap V_j = \emptyset
\]
\[
\forall a ∈ \bigcup_{x ∈ X} dom(x), \exists V ∈ \mathcal{V} : a ∈ V_i
\]

Remark 26 We have just seen that turning an element `<list>` into an element `<set>` allows us to define a variant for the constraint `sumCosts`. This principle will be generalized in Chapter 7.

4.1.5 Connection Constraints

In this section, we present constraints that establish a connection between different variables. More specifically, we introduce:

1. maximum
2. minimum
3. element
4. channel
5. permutation
6. precedence
4.1.5.1 Constraint maximum

The constraint maximum ensures that the maximum value among those assigned to variables of <list> respects a numerical condition.

**Syntax 52**

```xml
<maximum>
  <list (intVar wspace)2+</list>
  <condition> "(" operator "," operand ")" </condition>
</maximum>
```

**Semantics 23**

\[
\text{maximum}(X, (\odot, k)), \text{ with } X = (x_1, x_2, \ldots), \text{ iff } \\
\max\{x_i : 1 \leq i \leq |X| \odot k
\]

In the following example, the constraint \(c_1\) states that \(\max\{x_1, x_2, x_3, x_4\} = y\) whereas \(c_2\) states that \(\max\{z_1, z_2, z_3, z_4, z_5\} < w\).

**Example 62**

```xml
<maximum id="c1">
  <list> x1 x2 x3 x4 </list>
  <condition> (eq, y) </condition>
</maximum>
<maximum id="c2">
  <list> z1 z2 z3 z4 z5 </list>
  <condition> (lt, w) </condition>
</maximum>
```

Another form of maximum, sometimes called arg\_max, ensures that <index> is the index of a variable in <list> that respect the numerical condition. The optional attribute `startIndex` of <list> gives the number used for indexing the first variable in <list> (0, by default). The optional attribute `rank` of <index> indicates if <index> is the first index, the last index or any index of a variable of <list> that respect the numerical condition ("any", by default). The element <condition> becomes optional.

**Syntax 53**

```xml
<maximum>
  <list [startIndex=integer] (intVar wspace)2+ </list>
  <index [rank=rankType] intVar </index>
  [ <condition> "(" operator "," operand ")" </condition> ]
</maximum>
```

We give the semantics for rank="any".
Semantics 24

\[
\text{maximum}(X, i), \text{ with } X = \langle x_1, x_2, \ldots \rangle, \text{ iff } \\
\quad i \in \{ j : 1 \leq j \leq |X| \wedge x_j = \max\{x_k : 1 \leq k \leq |X|\}\} \\
\text{maximum}(X, i, (\odot, k)) \iff \text{maximum}(X, i) \wedge \text{maximum}(X, (\odot, k))
\]

4.1.5.2 Constraint \text{minimum}

The constraint \text{minimum} ensures that the minimum value among the values assigned to variables in \texttt{<list>} respects a numerical condition.

Syntax 54

\[
<\text{minimum}>
\begin{align*}
&<\text{list}> (\text{intVar wspace})^+ <\text{list}> \\
&<\text{condition}> "(" \text{operator } "," \text{operand } ")" <\text{condition}>
\end{align*}
<\text{minimum}>
\]

Semantics 25

\[
\text{minimum}(X, (\odot, k)), \text{ with } X = \langle x_1, x_2, \ldots \rangle, \text{ iff } \\
\quad \min\{x_i : 1 \leq i \leq |X|\} \odot k
\]

In the following example, the constraint \(c_1\) states that \(\min\{x_1, x_2, x_3, x_4\} = y\) whereas \(c_2\) states that \(\min\{z_1, z_2, z_3, z_4, z_5\} \neq w\).

Example 63

\[
<\text{minimum id="c1"}>
\begin{align*}
&<\text{list}> x_1 \ x_2 \ x_3 \ x_4 <\text{list}> \\
&<\text{condition}> (\text{eq}, y) <\text{condition}>
\end{align*}
<\text{minimum}>
<\text{minimum id="c2"}>
\begin{align*}
&<\text{list}> z_1 \ z_2 \ z_3 \ z_4 \ z_5 <\text{list}> \\
&<\text{condition}> (\text{ne}, w) <\text{condition}>
\end{align*}
<\text{minimum}>
\]

Another form of \text{minimum}, sometimes called \text{arg_min}, ensures that \texttt{<index>} is the index of the variable in \texttt{<list>} that respect the numerical condition. The optional attribute \text{startIndex} of \texttt{<list>} gives the number used for indexing the first variable in \texttt{<list>} (0, by default). The optional attribute \text{rank} of \texttt{<index>} indicates if \texttt{<index>} is the first index, the last index or any index of a variable of \texttt{<list>} that respect the numerical condition ("any", by default). The element \texttt{<condition>} becomes optional.
We give the semantics for rank="any".

Semantics 26
minimum(X,i), with X = \langle x_1, x_2, ... \rangle, iff // indexing assumed to start at 1
i \in \{ j : 1 \leq j \leq |X| \wedge x_j = \min \{ x_k : 1 \leq k \leq |X| \} \}
minimum(X,i,(\odot,k)) iff minimum(X,i) \wedge minimum(X,(\odot,k))

4.1.5.3 Constraint element
The constraint element [65] ensures that <value> is element of <list>, i.e., equal to one value among those assigned to variables of <list>. The optional attribute startIndex gives the number used for indexing the first variable in <list> (0, by default). The optional element <index> gives the index (position) of one occurrence of <value> inside <list>. Note that when the element <index> is absent, necessarily the attribute startIndex must also be absent (since it becomes irrelevant). The optional attribute rank of <index> indicates if <index> is the first index, the last index or any index of a variable of <list> with the specified value ("any", by default).

Syntax 56
<element>
  <list [startIndex="integer"]> (intVar wspace)2+ </list>
  [ <index [rank="rankType"]> intVar </index> ]
  <value> intVal | intVar </value>
</element>

We give the semantics for rank="any".

Semantics 27
element(X,v), with X = \langle x_1, x_2, ... \rangle, iff // indexing assumed to start at 1
\exists i : 1 \leq i \leq |X| \wedge x_i = v
element(X,i,v), with X = \langle x_1, x_2, ... \rangle, iff
x_i = v

The following constraint c_1 states that the ith variable of \langle x_1, x_2, x_3, x_4 \rangle must be equal to v. For example, if i is equal to 2, then x_2 must be equal to v. The constraint c_2 ensures that z is element (member) of the 1-dimensional array y.
A variant of the element considers that the element `<list>` only contain values (instead of variables). In that case, both elements `<index>` and `<value>` are required and must contain a variable. Although any such constraint can be reformulated as a binary extensional constraint, this variant is often used when modeling.

The semantics is:

\[
\text{element}(V, i, v), \text{ with } V = \langle v_1, v_2, \ldots \rangle, \text{ iff } v_i = v
\]

As an example, the constraint `c3` below ensures that, indexing starting at 0, if `i` is equal to 0 then `v` must be equal to 10, if `i` is equal to 1 then `v` must be equal to 4, and so on.

4.1.5.4 Constraint channel

The constraint `channel` ensures that if the `ith` variable is assigned the value `j`, then the `jth` variable must be assigned the value `i`. The optional attribute `startIndex` of `<list>` gives the number used for indexing the first variable in this list (0, by default).
Note that the opening and closing tags of \texttt{<list>} are optional when, of course, the attribute \texttt{startIndex} is not required, which gives:

\begin{verbatim}
Syntax 59
<channel> (intVar \texttt{wspace})2+ </channel>  // Simplified Form
\end{verbatim}

Semantics 29
\[
\text{channel}(X), \text{ with } X = (x_1, x_2, \ldots), \text{ iff } \forall i: 1 \leq i \leq |X|, x_i = j \Rightarrow x_j = i
\]

Note that it is possible to enforce a stronger form through the restriction "irreflexive"; see Section 7.3.

Another classical form of \texttt{channel}, sometimes called \texttt{inverse} or \texttt{assignment} in the literature, is defined from two separate lists of variables (that must be of same size). It ensures that the value assigned to the \textit{i}th variable of the first element \texttt{<list>} gives the position of the variable of the second element \texttt{<list>} that is assigned to \textit{i}, and vice versa. For each list, the optional attribute \texttt{startIndex} gives the number used for indexing the first variable in this list (0, by default).

\begin{verbatim}
Syntax 60
<channel>
<list [startIndex="integer"]> (intVar \texttt{wspace})2+ </list>
<list [startIndex="integer"]> (intVar \texttt{wspace})2+ </list>
</channel>
\end{verbatim}

Semantics 30
\[
\text{channel}(X, Y), \text{ with } X = (x_1, x_2, \ldots) \text{ and } Y = (y_1, y_2, \ldots), \text{ iff } \\
\forall i: 1 \leq i \leq |X|, x_i = j \Leftrightarrow y_j = i
\]

\textit{Condition}: $2 \leq |X| = |Y|$

\begin{verbatim}
Example 66
<channel id="c1">
<list> x1 x2 x3 x4 </list>
<list> y1 y2 y3 y4 </list>
</channel>
\end{verbatim}
It is possible to use this form of channel, with two lists of different sizes. The constraint then imposes restrictions on all variables of the first list, but not on all variables of the second list. The syntax is the same, but the semantics is the following (note that the equivalence has been replaced by an implication):

**Semantics 31**

\[
\text{channel}(X,Y), \text{ with } X = \langle x_1, x_2, \ldots \rangle \text{ and } Y = \langle y_1, y_2, \ldots \rangle, \text{ iff } \\
\forall i: 1 \leq i \leq |X|, x_i = j \Rightarrow y_j = i
\]

*Condition:* \(2 \leq |X| < |Y|\)

Another form is obtained by considering a list of 0/1 variables to be channeled with an integer variable. This third form of constraint channel ensures that the only variable of \(<\text{list}>\) that is assigned to 1 is at an index (position) that corresponds to the value assigned to the variable in \(<\text{value}>\).

**Syntax 61**

\[
<\text{channel}> \\
<\text{list} [\text{startIndex}=\text{integer}]> (\text{O1Var wspace})2+ </\text{list}> \\
<\text{value}> \text{intVar} </\text{value}> \\
</\text{channel}>
\]

**Semantics 32**

\[
\text{channel}(X,v), \text{ with } X = \{x_1, x_2, \ldots\}, \text{ iff } // \text{indexing assumed to start at 1} \\
\forall i: 1 \leq i \leq |X|, x_i = 1 \Leftrightarrow v = i \\
\exists i: 1 \leq i \leq |X| \land x_i = 1
\]

**Example 67**

\[
<\text{channel id="c2">} \\
<\text{list}> z1 z2 z3 z4 z5 </\text{list}> \\
<\text{value}> v </\text{value}> \\
</\text{channel}>
\]

### 4.1.5.5 Constraint permutation

The constraint permutation ensures that the tuple of values taken by variables of the second element \(<\text{list}>\) is a permutation of the tuple of values taken by variables of the first element \(<\text{list}>\). In other words, both lists represent the same multi-set. The optional element \(<\text{mapping}>\) gives for each value of the first tuple its position in the second tuple. If the element \(<\text{mapping}>\) is present, then it is possible to introduce the optional attribute startIndex that gives the number used for indexing the first variable in the second element \(<\text{list}>\) (0, by default).
Syntax 62

```xml
<permutation>
  <list> (intVar wspace)2+ </list>
  <list [startIndex="integer"]> (intVar wspace)2+ </list>
  [<mapping> (intVar wspace)2+ </mapping>]
</permutation>
```

Semantics 33

permutation($X, Y$), with $X = \langle x_1, x_2, \ldots \rangle$ and $Y = \langle y_1, y_2, \ldots \rangle$, iff

$\{|x_i : 1 \leq i \leq |X|\}| = \{|y_i : 1 \leq i \leq |Y|\}$

permutation($X, Y, M$), with $M = \langle m_1, m_2, \ldots \rangle$, iff

permutation($X, Y$) $\land \forall i : 1 \leq i \leq |X|, x_i = y_{m_i} \land \forall (i, j) : 1 \leq i < j \leq |M|, m_i \neq m_j$

Prerequisite : $|X| = |Y| = |M| \geq 2$

Example 68

```xml
<permutation id="c">
  <list> x1 x2 x3 y4 </list>
</permutation>
```

The constraint permutation captures correspondence in the Global Constraint Catalog. For a generalization of permutation, see allEqual on multisets in Section 7.1.2. For a specialization of permutation, where the second list is increasingly sorted, see Section 7.3.

4.1.5.6 Constraint precedence

The constraint precedence, see [76, 142], ensures that if a variable $x$ of <list> is assigned the $i + 1$th value of <values>, then another variable of <list>, that precedes $x$, is assigned the $i$th value of <values>. The optional attribute covered indicates whether each value of <values> must be assigned by at least one variable in <list> ("false", by default). It is also possible to reverse the order by specifying the relational operator in the optional element <operator> (lt, by default).

Syntax 63

```xml
<precedence>
  <list> (intVar wspace)2+ </list>
  <values [covered="boolean"]> (intVal wspace)2+ </values>
  [operator] "lt" | "gt" </operator>
</precedence>
```

For the semantics, $V^{cv}$ means covered="true".

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Semantics 34

\[ \text{precedence}(X, V, \odot), \text{ with } X = \langle x_1, x_2, \ldots \rangle, V = \langle v_1, v_2, \ldots \rangle, \text{ and } \odot \in \{<, >\} \text{ iff} \]
\[ \forall i : 1 \leq i < |V|, v_{i+1} \in \{x_i : 1 \leq i \leq |X|\} \Rightarrow v_i \in \{x_i : 1 \leq i \leq |X|\} \]
\[ \forall i : 1 \leq i < |V| \land v_{i+1} \in \{x_i : 1 \leq i \leq |X|\}, \]
\[ \min\{j : 1 \leq j \leq |X| \land x_j = v_i\} \odot \min\{j : 1 \leq j \leq |X| \land x_j = v_{i+1}\} \]
\[ \text{precedence}(X, V^\text{cv}, \odot) \text{ iff precedence}(X, V, \odot) \land v_{|V|} \in \{x_i : 1 \leq i \leq |X|\} \]

Example 69

\[ \text{precedence} \]
\[ \langle \text{list} \rangle x1 \ x2 \ x3 \ x4 \ </\text{list}> \]
\[ \langle \text{values} \rangle 4 \ 0 \ 1 \ </\text{values}> \]
\[ </\text{precedence}> \]

The constraint precedence captures `int_value_precede` and `int_value_precede_chain` in the Global Constraint Catalog.

4.1.6 Packing and Scheduling Constraints

In this section, we present constraints that are useful in packing and scheduling problems. More specifically, we introduce:

1. `stretch`
2. `noOverlap` (capturing disjunctive and diffn)
3. `cumulative` (capturing cumulatives too)
4. `binPacking`
5. `knapsack`
6. `networkFlow`

4.1.6.1 Constraint stretch

The constraint `stretch` [99] aims at grouping values in sequences. Each stretch is a maximal subsequence with the same value over the list of variables in `<list>`. Each stretch must have a width compatible with the corresponding interval specified in `<widths>`. The optional element `<patterns>` gives the possible successive values between stretches.

Syntax 64

\[ \langle \text{stretch} \rangle \]
\[ \langle \text{list} \rangle (\text{intVar wspace})2+ \ </\text{list}> \]
\[ \langle \text{values} \rangle (\text{intVal wspace})+ \ </\text{values}> \]
\[ \langle \text{widths} \rangle (\text{intIntvl wspace})+ \ </\text{widths}> \]
\[ \langle \text{patterns} \rangle ("\"\text{intVal }\","\text{intVal }\")+ \ </\text{patterns}> \]
\[ </\text{stretch}> \]

Given a sequence of variables \( X = \langle x_1, x_2, \ldots \rangle \), and a value assigned to each of these variables, we define as stretches(\( X \)) the set of stretches over \( X \), each one being identified by
a pair \((v_k, i..j)\) composed of the value \(v_k\) of the stretch and the interval \(i..j\) of the indexes of the variables composing the stretch.

**Semantics 35**

\[
\text{stretch}(X, V, W), \text{ with } X = \langle x_1, x_2, \ldots \rangle, \ V = \langle v_1, v_2, \ldots \rangle, \ W = \langle l_1, l_2, \ldots \rangle, \text{ iff }
\forall (v_k, i..j) \in \text{stretches}(X), l_k \leq j - i + 1 \leq u_k
\]

\[
\text{stretch}(X, V, W, P) \text{ iff }
\text{stretch}(X, V, W) \land \forall i : 1 \leq i < |X|, x_i \neq x_{i+1} \Rightarrow (x_i, x_{i+1}) \in P
\]

*Prerequisite*: \(|V| = |W|

Although the section is devoted to integer variables, we give an example with symbolic values, as this is a straightforward adaptation.

**Example 70**

```xml
<stretch>
  <list> x1 x2 x3 x4 x5 x6 x7 </list>
  <values> morning afternoon night off </values>
  <widths> 1..3 1..3 2..3 2..4 </widths>
</stretch>
```

**Remark 27** *In the future, we might authorize longer forms of patterns.*

### 4.1.6.2 Constraint noOverlap

We start with the one dimensional form of noOverlap [67] that corresponds to *disjunctive* [30] and ensures that some tasks, defined by their origins and durations (lengths), must not overlap. The attribute *zeroIgnored* is optional ("true", by default): when set to "false", it indicates that zero-length tasks cannot be packed anywhere (cannot overlap with other tasks).

**Syntax 65**

```xml
<noOverlap [zeroIgnored="boolean"] >
  <origins> (intVar wspace)2+ </origins>
  <lengths> (intVal wspace)2+ | (intVar wspace)2+ </lengths>
</noOverlap>
```

The semantics is given for *zeroIgnored="false"*

**Semantics 36**

\[
\text{noOverlap}(X, L), \text{ with } X = \langle x_1, x_2, \ldots \rangle \text{ and } L = \langle l_1, l_2, \ldots \rangle, \text{ iff }
\forall (i, j) : 1 \leq i < j \leq |X|, x_i + l_i \leq x_j \lor x_j + l_j \leq x_i
\]

*Prerequisite*: \(|X| = |L| \geq 2*
The k-dimensional form of `noOverlap` corresponds to `diffn [8]` and ensures that, given a set of n-dimensional boxes; for any pair of such boxes, there exists at least one dimension where one box is after the other, i.e., the boxes do not overlap. The attribute `zeroIgnored` is optional ("true", by default); when set to "false", it indicates that zero-width boxes cannot be packed anywhere (cannot overlap with other boxes).

The semantics is given for `zeroIgnored="false"`. The following constraint enforces that all four 3-dimensional specified boxes do not overlap. The first box has origin \((x_1, y_1, z_1)\) and length \((2, 4, 1)\), meaning that this box is situated from \(x_1\) to \(x_1 + 2\) on x-axis, from \(y_1\) to \(y_1 + 4\) on y-axis, and from \(z_1\) to \(z_1 + 1\) on z-axis.

**Example 72**

```xml
<noOverlap>
  <origins> (x1,y1,z1)(x2,y2,z2)(x3,y3,z3)(x4,y4,z4) </origins>
  <lengths> (2,4,1)(4,2,3)(5,1,2)(3,3,2) </lengths>
</noOverlap>
```

### 4.1.6.3 Constraint cumulative

Here, we have a collection of tasks, each one described by 4 attributes: origin, length, end and height. In XCSF3, the element `<ends>` is optional. The constraint `cumulative [1]` enforces that at each point in time, the cumulated height of tasks that overlap that point, respects a
numerical condition. When the operator “le” is used, this corresponds to not exceeding a given limit.

**Syntax 67**

```xml
<cumulative>
  <origins> (intVar wspace)2+ </origins>
  <lengths> (intVal wspace)2+ | (intVar wspace)2+ </lengths>
  [ <ends> (intVar wspace)2+ </ends> ]
  <heights> (intVal wspace)2+ | (intVar wspace)2+ </heights>
  <condition> "(" operator "," operand ")" </condition>
</cumulative>
```

**Semantics 38**

`cumulative(X, L, H, ⟨⊙, k⟩), with X = ⟨x₁, x₂, ...⟩, L = ⟨l₁, l₂, ...⟩, H = ⟨h₁, h₂, ...⟩, iff

∀t ∈ N, \( \sum\{h_i : 1 \leq i \leq |H| \land x_i \leq t < x_i + l_i\} \odot k \)

Prerequisite: \(|X| = |L| = |H| \geq 2

If the element `<ends>` is present, for the semantics, we have additionally that:

\( \forall i : 1 \leq i \leq |X|, x_i + l_i = e_i \)

**Example 73**

```xml
<cumulative>
  <origins> s1 s2 s3 s4 </origins>
  <lengths> 11 12 13 14 </lengths>
  <heights> h1 h2 h3 h4 </heights>
  <condition> (le,4) </condition>
</cumulative>
```

A refined form of `cumulative` corresponds to the constraint sometimes called `cumulatives` [5]. We have a set of available machines (resources), with their respective numerical conditions in `<conditions>`, which replaces `<condition>`, through a sequence of pairs composed of an operator and an operand. Each task is thus performed by a machine, which is given in an element `<machines>`. For each machine, the cumulated height of tasks that overlap a time point for this machine must respect the numerical (capacity) condition associated with the machine. The optional attribute `startIndex` gives the number used for indexing the first numerical condition (and so, the first machine) in this list (0, by default).

**Syntax 68**

```xml
<cumulative>
  <origins> (intVar wspace)2+ </origins>
  <lengths> (intVal wspace)2+ | (intVar wspace)2+ </lengths>
  [ <ends> (intVar wspace)2+ </ends> ]
  <heights> (intVal wspace)2+ | (intVar wspace)2+ </heights>
</cumulative>
```
Semantics 39

\[
\text{cumulative}(X, L, H, M, C), \text{ with } X = \langle x_1, x_2, \ldots \rangle, \ L = \langle l_1, l_2, \ldots \rangle, \ H = \langle h_1, h_2, \ldots \rangle, \ M = \langle m_1, m_2, \ldots \rangle, \text{ and } C = ((\odot_1, k_1), (\odot_2, k_2), \ldots)
\]

iff

\[\forall m \in \{m_i : 1 \leq i \leq |M|\}, \forall t \in \mathbb{N} : \exists 1 \leq i \leq |H| : x_i \leq t < x_i + l_i \land m_i = m, \sum_{1 \leq i \leq |H|} \{h_i : 1 \leq i \leq |H| \land x_i \leq t < x_i + l_i \land m_i = m\} \odot m k_m\]

Prerequisite: \(|X| = |L| = |H| = |M| \geq 2\)

4.1.6.4 Constraint binPacking

The constraint binPacking [127, 122, 29] ensures that a list of items, whose sizes are given, are put in different bins in such a way that the total size of the items in each bin respects a numerical condition (always the same, because the capacity is assumed to be the same for all bins). When the operator “le” is used, this corresponds to not exceeding the capacity of each bin.

Syntax 69

<binPacking>
  <list> (intVar wspace)2+ </list>
  <sizes> (intVal wspace)2+ </sizes>
  <condition> "( operator "," operand ")" </condition>
</binPacking>

Semantics 40

\[
\text{binPacking}(X, S, (\odot, k)), \text{ with } X = \langle x_1, x_2, \ldots \rangle \text{ and } S = \langle s_1, s_2, \ldots \rangle, \text{ iff}
\]

\[\forall b \in \{x_i : 1 \leq i \leq |X|\}, \sum_{1 \leq i \leq |S|} \{s_i : 1 \leq i \leq |S| \land x_i = b\} \odot k\]

Prerequisite: \(|X| = |S| \geq 2\)

Example 74

<binPacking id="c1">
  <list> x1 x2 x3 x4 x5 </list>
  <sizes> 25 53 38 41 32 </sizes>
  <condition> (1e,100) </condition>
</binPacking>
In another form of *binPacking*, a specific numerical condition is associated with each available bin. The element `<condition>` is replaced by an element `<conditions>`. The optional attribute `startIndex` gives the number used for indexing the first numerical condition (and so, the first bin) in this list (0, by default).

**Syntax 70**

```xml
<binPacking>
  <list> (intVar wspace)2+ </list>
  <sizes> (intVal wspace)2+ </sizes>
  <conditions [startIndex="integer"]>
    ("(" operator "," operand ")")2+
  </conditions>
</binPacking>
```

For the semantics, we assume that `startIndex="1"`.

**Semantics 41**

\[
\text{binPacking}(X, S, C), \text{ with } X = \langle x_1, x_2, \ldots \rangle, \ S = \langle s_1, s_2, \ldots \rangle, \text{ and } C = \langle ((\circ_1, k_1), (\circ_2, k_2), \ldots \rangle, \text{ iff } \forall b: 1 \leq b \leq |C|, \sum \{s_i : 1 \leq i \leq |S| \land x_i = b\} \circ_b k_b
\]

**Prerequisite**: \(|X| = |S| \geq 2 \land |C| \geq 2\)

The constraint \(c_2\) involves 7 items and 3 bins. Here, the variables \(l_1\), \(l_2\) and \(l_3\) are used to denote loads (since the operator is systematically “eq”).

**Example 75**

```xml
<binPacking id="c2">
  <list> y1 y2 y3 y4 y5 y6 y7 </list>
  <sizes> 12 7 21 36 19 22 30 </sizes>
  <conditions> (eq,11)(eq,12)(eq,13) </conditions>
</binPacking>
```

### 4.1.6.5 Constraint knapsack

The constraint *knapsack* \([49, 125, 91]\) ensures that some items are packed in a knapsack with certain weight and profit restrictions.

**Syntax 71**

```xml
<knapack>
  <list> (intVar wspace)2+ </list>
  <weights> (intVal wspace)2+ </weights>
  <profits> (intVal wspace)2+ </profits>
  <limit> intVal | intVar </limit>
  <condition> "(" operator "," operand ")" </condition>
</knapack>
```
**Semantics 42**

**knapsack** \((X, W, P, w, (\odot, k))\), with \(X = (x_1, x_2, \ldots)\), \(W = (w_1, w_2, \ldots)\), and \(P = (p_1, p_2, \ldots)\), iff

\[
\sum_{i=1}^{\|X\|} (w_i \times x_i) \leq w \\
\sum_{i=1}^{\|X\|} (p_i \times x_i) \odot k
\]

**Prerequisite**: \(|X| = |W| = |P| \geq 2\)

### 4.1.6.6 Constraint networkFlow

The constraint **networkFlow** ensures that there is a valid flow in a graph. The flow of each arc and the balance (difference between input and output flows) of each node are given by elements **<list>** and **<balance>**, respectively. Optionally, a unit weight can be associated with each arc, and the total cost must then be related to the overall cost of the flow by a numerical condition. Also, there can be an element **<capacities>** that give lower and upper capacities of each arc.

**Syntax 72**

```xml
<networkFlow>
  <list> (intVar wspace)2+ </list>
  <balance> (intVal wspace)2+ | (intVar wspace)2+ </balance>
  <arcs> (" node "," node ")2+ </arcs>
  [[<capacities> (intIntvl wspace)2+ </capacities>] 
    [ <weights> (intVal wspace)2+ | (intVar wspace)2+ </weights> 
      <condition> (" operator "," operand ")" </condition>
    ]
  </networkFlow>
```

For the semantics, considering the graph defined by **<arcs>**, for any node \(v\), \(\Gamma^+(v)\) denotes the set of arcs (of this graph) with head \(v\) and \(\Gamma^-(v)\) denotes the set of arcs with tail \(v\).

**Semantics 43**

**networkFlow**\((X, B, A)\), with \(X = (x_1, x_2, \ldots)\), \(B = (b_1, b_2, \ldots)\), \(A = (a_1, a_2, \ldots)\), iff

for any node \(v\),

\[
\sum\{x_i : 1 \leq i \leq |X| \land a_i \in \Gamma^-(v)\} - \sum\{x_i : 1 \leq i \leq |X| \land a_i \in \Gamma^+(v)\} = b_i
\]

**networkFlow**\((X, B, A, C)\), with \(C = (l_1, u_1, l_2, u_2, \ldots)\), iff

\[
\sum\{x_i : 1 \leq i \leq |X|, x_i \in l_i..u_i\}
\]

**networkFlow**\((X, B, A, C, W, (\odot, k))\), with \(W = (w_1, w_2, \ldots)\), iff

\[
\sum\{x_i \times w_i : 1 \leq i \leq |X|\} \odot k
\]

**Prerequisite**: \(|X| = |A| = |C| = |W|\)

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4.1.7 Constraints on Graphs

In this section, we present constraints that are defined on graphs using integer variables (encoding called “successors variables”). We introduce:

1. circuit
2. nCircuits
3. path
4. nPaths
5. tree
6. nTrees

For all these constraints, we have an element <list> that contains variables \( x_1, x_2, \ldots \). The assumption is that each pair \( (i, x_i) \) represents an arc (or edge) of the graph to be built; if \( x_i = j \), then it means that the successor of node \( i \) is node \( j \). Note that a loop (also called self-loop) corresponds to a variable \( x_i \) such that \( x_i = i \); it is isolated if there is no variable \( x_j \) with \( j \neq i \) such that \( x_j = i \).

4.1.7.1 Constraint circuit

The constraint circuit [8] ensures that the values taken by the variables in <list> forms a circuit, with the assumption that each pair \( (i, x_i) \) represents an arc. The optional attribute startIndex of <list> gives the number used for indexing the first variable/node in <list> (0, by default). The optional element size indicates that the circuit must be of a given size (strictly greater than 1).

It is important to note that the circuit is not required to cover all nodes (the nodes that are not present in the circuit are then self-looping). Hence circuit, with loops being simply ignored, basically represents subcircuit (e.g., in MiniZinc). If ever you need a full circuit (i.e., without any loop), you have three solutions:

- indicate in size the number of variables
- initially define the variables without the self-looping values,
- post unary constraints.

**Syntax 73**

```
<circuit>
  <list [startIndex="integer"]> (intVar wspase)2+ </list>
  [<size> intVal | intVar </size>]
</circuit>
```

Note that the opening and closing tags of <list> are optional if <list> is the unique parameter of the constraint and the attribute startIndex is not necessary, which gives:

**Syntax 74**

```
<circuit> (intVar wspase)2+ </circuit> // Simplified Form
```
In the following example, the constraint states that \( \langle x_0, x_1, x_2, x_3 \rangle \) must form a single circuit – for example, with \( \langle x_0, x_1, x_2, x_3 \rangle = \langle 1, 2, 3, 0 \rangle \), or a subcircuit – for example, with \( \langle x_0, x_1, x_2, x_3 \rangle = \langle 1, 2, 0, 3 \rangle \).

Example 76

\[
\begin{align*}
\text{<circuit>}
\quad x_0 \ x_1 \ x_2 \ x_3 \\
\end{align*}
\]

Note that it is possible to capture the variant of \texttt{circuit} in Gecode that involves a cost matrix. It suffices to logically combine \texttt{circuit} and \texttt{sumCosts} as shown in Section 8.4.

4.1.7.2 Constraint \texttt{nCircuits}

One may be interested in building more than one circuit. For constraint \texttt{nCircuits}, the number of circuits obtained after instantiating variables of \texttt{<list>} must respect a numerical condition. In the literature, this variant with \( k > 1 \) circuits is called \texttt{cycle}. The optional attribute \texttt{countLoops} indicates whether or not loops must be considered as circuits ("true", by default).

Syntax 75

\[
\begin{align*}
\text{<nCircuits [countLoops="boolean"]>}
\quad \text{<list [startIndex="integer"]> (intVar wspace)2+ </list>}
\quad \text{<condition> "(" operator "," operand ")" </condition>}
\quad \text{</nCircuits>}
\end{align*}
\]

Semantics 45

\[
\begin{align*}
n\text{Circuits}(X,(\odot,k)), \text{ with } X = \langle x_1, x_2, \ldots \rangle, \text{ iff } \\
\quad \{(i, x_i) : 1 \leq i \leq |X| \land i \neq x_i\} \text{ forms } p \text{ node-disjoint circuits s.t. } p \odot k
\end{align*}
\]

Remark 28 It is always possible to avoid having self-loops as circuits, by combining \texttt{circuits} with a set of unary constraints.

Below, the constraint states that \( \langle y_0, y_1, y_2, y_3, y_4 \rangle \) must form two disjoint circuits – for example, with \( \langle y_0, y_1, y_2, y_3, y_4 \rangle = \langle 1, 0, 4, 2, 3 \rangle \).
4.1.7.3 Constraint path

The constraint path ensures that the values taken by the variables in <list> forms a path from the node specified in <start> to the node specified in <final>, with the assumption that each pair \((i, x_i)\) represents an arc. The optional attribute startIndex of <list> gives the number used for indexing the first variable/node in <list> (0, by default). The successor of the final node is necessary itself (self-loop). The optional element size indicates that the path must be of a given size.

It is important to note that the path is not required to cover all nodes (the nodes that are not present in the path are then self-looping). Hence path, with loops being simply ignored, basically represents subpath (e.g., in Choco3). If ever you need a full path (i.e., without any loop other than the final node), you have three solutions:

- indicate in size the number of variables
- initially define the variables without the self-looping values,
- post unary constraints.

Syntax 76

```xml
<path>
  <list [startIndex="integer"]> (intVar wspace)2+ </list>
  <start> intVal | intVar </start>
  <final> intVal | intVar </final>
  [<size> intVal | intVar </size>]
</path>
```

Semantics 46

```latex
\text{path}(X, s, f), \text{ with } X = \langle x_1, x_2, \ldots \rangle, \text{ iff } \{ (i,x_i) : 1 \leq i \leq |X| \land i \neq x_i \} \text{ forms a path from } s \text{ to } f \\
\text{path}(X, s, f, z), \text{ with } X = \langle x_1, x_2, \ldots \rangle, \text{ iff } \{ (i,x_i) : 1 \leq i \leq |X| \land i \neq x_i \} \text{ forms a path from } s \text{ to } f \text{ of size } z
```

In the following example, the constraint states that \(\langle x_0, x_1, x_2, x_3 \rangle\) must form a path from \(s\) to \(f\) – for example, \(\langle x_0, x_1, x_2, x_3 \rangle = \langle 0, 0, 3, 1 \rangle\) with \(s = 2\) and \(f = 0\) satisfies the constraint.
4.1.7.4 Constraint \texttt{nPaths}

The constraint \texttt{nPaths} ensures that the number of paths formed after instantiating variables of \texttt{<list>} must respect a numerical condition. The optional attribute \texttt{countLoops} indicates whether or not isolated loops must be considered as paths ("true", by default).

\textbf{Syntax 77}

\begin{verbatim}
<nPaths [countLoops="boolean"]>
  <list [startIndex="integer"]> (intVar wspace)2+ </list>
  <condition> "(" operator "," operand ")" </condition>
</nPaths>
\end{verbatim}

\textbf{Semantics 47}

\texttt{nPaths}(X,(\circ,k)), with \(X = \langle x_1, x_2, \ldots \rangle\), iff

\[ \{ (i, x_i) : 1 \leq i \leq |X| \} \text{ forms } p \text{ node-disjoint paths s.t. } p \circ k \]

The constraint \(c\) states that \(\langle y_0, y_1, \ldots, y_7 \rangle\) must form \(k\) disjoints paths – for example, \(\langle y_0, y_1, \ldots, y_7 \rangle = \langle 0, 2, 4, 6, 0, 5, 6, 5 \rangle\) with \(k = 3\).

\textbf{Example 79}

\begin{verbatim}
<nPaths id="c">
  <list> y0 y1 y2 y3 y4 y5 y6 y7 </list>
  <condition> (eq,k) </condition>
</nPaths>
\end{verbatim}

4.1.7.5 Constraint \texttt{tree}

The constraint \texttt{tree} [9, 48] ensures that the values taken by variables in \texttt{<list>} forms an anti-arborescence (covering all nodes) whose root is specified by \texttt{<root>}, with the assumption that each pair \((i, x_i)\) represents an arc (with such representation, we do obtain an anti-arborescence).

The optional attribute \texttt{startIndex} of \texttt{<list>} gives the number used for indexing the first variable/node in \texttt{<list>} (0, by default). The successor of the root node is necessarily itself (self-loop). The optional element \texttt{size} indicates that the anti-arborescence must be of a given size.

It is important to note that the anti-arborescence is not required to cover all nodes (the nodes that are not present in the anti-arborescence are then self-looping). Hence \texttt{tree}, with
loops being simply ignored, basically represents subtree. If ever you need a full anti-arborescence (i.e., without any loop other than the root node), you have three solutions:

- indicate in size the number of variables
- initially define the variables without the self-looping values,
- post unary constraints.

**Syntax 78**

```
<tree>
  <list [startIndex="integer"]> (intVar wspace)2+ </list>
  <root> intVal | intVar </root>
  [<size> intVal | intVar </size>]
</tree>
```

**Semantics 48**

\[
\text{tree}(X,r), \text{ with } X = (x_1, x_2, \ldots), \text{ iff}
\{ (i, x_i) : 1 \leq i \leq |X| \land i \neq x_i \} \text{ forms an anti-arborescence of root } r.
\]

\[
\text{tree}(X,r,z), \text{ with } X = (x_1, x_2, \ldots), \text{ iff}
\{ (i, x_i) : 1 \leq i \leq |X| \land i \neq x_i \} \text{ forms an anti-arborescence of root } r \text{ and size } z
\]

In the following example, the constraint states that \(x_0, x_1, x_2, x_3\) must form an anti-arborescence of root \(r\) – for example, \(x_0, x_1, x_2, x_3 = (1, 3, 3, 3)\) with \(r = 3\) satisfies the constraint.

**Example 80**

```
<tree>
  <list >x0 x1 x2 x3 </list>
  <root > r </root>
</tree>
```

4.1.7.6 Constraint nTrees

The constraint nTrees ensures that the number of anti-arborescences formed after instantiating variables in <list> must respect a numerical condition. The optional attribute countLoops indicates whether or not isolated loops must be considered as trees ("true", by default).

**Syntax 79**

```
<nTrees [countLoops="boolean"]>
  <list> (intVar wspace)2+ </list>
  <condition> "(" operator "," operand ")" </condition>
</nTrees>
```
Semantics 49

\[ nTrees(X, (\odot, k)), \text{ with } X = (x_1, x_2, \ldots), \text{ iff } \{ (i, x_i) : 1 \leq i \leq |X| \} \text{ forms } p \text{ disjoint anti-arborescences s.t. } p \odot k \]

The constraint \( c \) states that \( \langle y_0, y_1, \ldots, y_7 \rangle \) must form \( k \) node-disjoint anti-arborescences – for example, \( \langle y_0, y_1, \ldots, y_7 \rangle = \langle 0, 4, 4, 6, 0, 6, 4 \rangle \) with \( k = 2 \).

Example 81

\[
\begin{align*}
\langle nTrees \text{ id="c"} > \\
\langle list \rangle & y_0 \ y_1 \ y_2 \ y_3 \ y_4 \ y_5 \ y_6 \ y_7 \langle/\list \rangle \\
\langle/condition \rangle \end{align*}
\]

4.1.8 Elementary Constraints

In this section, we present some elementary constraints that are frequently encountered. We introduce:

1. clause
2. instantiation

4.1.8.1 Constraint clause

The constraint \textit{clause} represents a disjunction of literals, put in an element \textit{<list>}, where a literal is either a 0/1 variable (serving as a Boolean variable) or its negation. The negation of a variable \( x \) is simply written \textit{not}(\( x \)).

Syntax 80

\[
\langle\text{clause}\rangle \\
\langle\text{list}\rangle (\langle\text{01Var}| \ "\text{not}"(\langle\text{01Var}\rangle)\rangle \ wspace)+ \langle/\text{list}\rangle \\
\langle/\text{clause}\rangle
\]

Note that the opening and closing tags of \textit{<list>} are optional, which gives:

Syntax 81

\[
\langle\text{clause}\rangle (\langle\text{01Var}| \ "\text{not}"(\langle\text{01Var}\rangle)\rangle \ wspace)+ \langle/\text{clause}\rangle \quad \text{// Simplified Form}
\]

Semantics 50

\[
\text{clause}(L) \text{ iff } \exists x \in L : x = 1 \lor \exists \neg x \in L : x = 0
\]

In the following example, the constraint states that \( x_0 \lor \neg x_1 \lor x_2 \lor x_3 \lor \neg x_4 \)
4.1.8.2 Constraint instantiation

The constraint instantiation represents a set of unary constraints corresponding to variable assignments. There are three main interests in introducing it. First, when modeling, rather often we need to instantiate a subset of variables (for example, for taking some hints/clues into consideration, or for breaking some symmetries). It is simpler, more natural and informative to post a constraint instantiation than a set of unary constraints intension. Second, instantiations can be used to represent partial search instantiations that can be directly injected into XCSP3 instances. Third, instantiations allow us to represent solutions, as explained in Section 2.11, having this way the output of the format made compatible with the input.

The constraint instantiation ensures that the ith variable \( x \) of \(<list>\) is assigned the ith value of \(<values>\). Every variable can only occur once in \(<list>\).

**Syntax 82**

```xml
<instantiation>
  <list> (intVar wspace)+ </list>
  <values> (intVal wspace)+ </values>
</instantiation>
```

**Semantics 51**

\[ \text{instantiation}(X, V), \text{with } X = (x_1, \ldots, x_r) \text{ and } V = (v_1, \ldots, v_r) \iff \forall i : 1 \leq i < |X|, x_i = v_i \]

In our example below, we have a first constraint that enforces \( x = 12, y = 4 \) and \( z = 30 \), and a second constraint involving an array \( w \) of 6 variables such that \( w[0] = 1, w[1] = 0, w[2] = 2, w[3] = 1, w[4] = 3, w[5] = 1 \).

**Example 83**

```xml
<instantiation>
  <list> x y z </list>
  <values> 12 4 30 </values>
</instantiation>
<instantiation>
  <list> w[] </list>
  <values> 1 0 2 1 3 1 </values>
</instantiation>
```

**Remark 29** The constraint instantiation is a generalization of the constraint cube used in the SAT community when dealing with DNF (Disjunctive Normal Form) expressions.
Remark 30  The attributes coming with the element <instantiation> when used to represent a solution (see Section 2.11) are simply ignored (and tolerated) when a solution is injected in the input.

4.2  Constraints over Symbolic Variables

Many constraints introduced above for integer variables can be used with symbolic variables. Basically, this is always when no arithmetic computation is involved. In the current version of this document, we shall not list exhaustively all such cases.

We just give an illustration of an extensional constraint involving three symbolic variables of domain \{a, b, c\}.

<extension id="c1">
  <list> x1 x2 x3 </list>
  <supports> (a,b,a)(b,a,c)(b,c,b)(c,a,b) </supports>
</extension>
In this chapter, we introduce constraints over complex discrete variables, namely, set and graph variables. We shall use the terms setVar and graphVar to denote respectively a set and a graph variable. These terms as well as related ones, like for example, setVal, are defined in Appendix B.

5.1 Constraints over Set Variables

We first introduce constraints over set variables per family, as indicated by Figure 5.1. In what follows, set variables are denoted by lowercase letters $s$, $t$, possibly subscripted. For sequences of set variables, we use uppercase letters $S$, $T$, possibly subscripted, as e.g., $S = (s_1, s_2, \ldots)$. For sequences of sequences of set variables, we use uppercase letters $S$, $T$ in mathcal font, as e.g., $S = (S_1, S_2, \ldots)$.

5.1.1 Generic Constraints

5.1.1.1 Constraint intension

Unsurprisingly, a constraint intension defined on set variables is built similarly to a constraint intension defined on integer variables. Simply, some additional operators are available. They are given by Table 5.1 that shows the specific operators that can be used to build predicates with set operands. Of course, it is possible to combine such operators with those presented for integers (and Booleans) in Table 4.1. Note that the operators on convexity are used in Gecode.

An intensional constraint is defined by an element <intension>, containing an element <function> that describes the functional representation of the predicate, referred to as bool-ExprSet in the syntax box below, and whose precise syntax is given in Appendix B.
Figure 5.1: The different types of constraints over set variables.

Syntax 83

```
<intension>
  <function> boolExprSet </function>
</intension>
```

Note that the opening and closing tags for `<function>` are optional, which gives:

Syntax 84

```
<intension> boolExprSet </intension> // Simplified Form
```

In the following example, the constraints $c_1$ and $c_2$ are respectively defined by $|s| = 2$ and $t_1 \cup t_2 \subset t_3$, with $s$, $t_1$, $t_2$ and $t_3$ being set variables.
### Table 5.1: Operators on sets that can be used to build predicates.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Arity</th>
<th>Syntax</th>
<th>Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic (set operands and integer result)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cardinality</td>
<td>1</td>
<td>$\text{card}(s)$</td>
<td>$</td>
</tr>
<tr>
<td>Smallest element</td>
<td>1</td>
<td>$\text{min}(s)$</td>
<td>$\text{min } s$</td>
</tr>
<tr>
<td>Greatest element</td>
<td>1</td>
<td>$\text{max}(s)$</td>
<td>$\text{max } s$</td>
</tr>
<tr>
<td>Set (set operands, except for the operator $\text{set}$, and set result)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{set}$</td>
<td>$r &gt; 0$</td>
<td>${x_1, \ldots, x_r}$</td>
<td>${x_1, \ldots, x_r}$</td>
</tr>
<tr>
<td>Union</td>
<td>$r \geq 2$</td>
<td>$\text{union}(s_1, \ldots, s_r)$</td>
<td>$s_1 \cup \ldots \cup s_r$</td>
</tr>
<tr>
<td>Intersection</td>
<td>$r \geq 2$</td>
<td>$\text{inter}(s_1, \ldots, s_r)$</td>
<td>$s_1 \cap \ldots \cap s_r$</td>
</tr>
<tr>
<td>Difference</td>
<td>2</td>
<td>$\text{diff}(s,t)$</td>
<td>$s \setminus t$</td>
</tr>
<tr>
<td>Symmetric difference</td>
<td>$r \geq 2$</td>
<td>$\text{sdiff}(s_1, \ldots, s_r)$</td>
<td>$s_1 \Delta \ldots \Delta s_r$</td>
</tr>
<tr>
<td>Convex Hull</td>
<td>1</td>
<td>$\text{hull}(s)$</td>
<td>${i : \text{min } s \leq i \leq \text{max } s}$</td>
</tr>
<tr>
<td>Relational (set operands, except for the 1st operand of the operator $\text{in}$, and Boolean result)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Membership</td>
<td>2</td>
<td>$\text{in}(x,s)$</td>
<td>$x \in s$</td>
</tr>
<tr>
<td>Different from</td>
<td>2</td>
<td>$\text{ne}(s,t)$</td>
<td>$s \neq t$</td>
</tr>
<tr>
<td>Equal to</td>
<td>$r \geq 2$</td>
<td>$\text{eq}(s_1, \ldots, s_r)$</td>
<td>$s_1 = \ldots = s_r$</td>
</tr>
<tr>
<td>Disjoint sets</td>
<td>2</td>
<td>$\text{djoint}(s,t)$</td>
<td>$s \cap t = \emptyset$</td>
</tr>
<tr>
<td>Strict subset</td>
<td>2</td>
<td>$\text{subset}(s,t)$</td>
<td>$s \subset t$</td>
</tr>
<tr>
<td>Subset or equal to</td>
<td>2</td>
<td>$\text{subseq}(s,t)$</td>
<td>$s \subseteq t$</td>
</tr>
<tr>
<td>Strict superset</td>
<td>2</td>
<td>$\text{supset}(s,t)$</td>
<td>$s \supset t$</td>
</tr>
<tr>
<td>Convexity</td>
<td>1</td>
<td>$\text{convex}(s)$</td>
<td>$s = {i : \text{min } s \leq i \leq \text{max } s}$</td>
</tr>
</tbody>
</table>

**Example 85**

```xml
<intension id="c1">
  eq(\text{card}(s),2)
</intension>

<intension id="c2">
  \text{subset}(\text{union}(t_1, t_2), t_3)
</intension>
```

### 5.1.2 Comparison-based Constraints

#### 5.1.2.1 Constraint allDifferent

Such constraints are similar to those described earlier for integer variables, but set variables/values simply replace integer variables/values.

**Syntax 85**

```xml
<allDifferent>
  <list> (setVar wspace)2+ </list>
  [<except> setVal+ </except>]
</allDifferent>
```
Note that the opening and closing tags for `<list>` are optional, if `<except>` is not present, which gives:

```
Syntax 86
| <allDifferent> (setVar wspace)2+ </allDifferent>           // Simplified Form

Example 86
| <allDifferent id="c1"> s1 s2 s3 </allDifferent>
| <allDifferent id="c2"> <list> t1 t2 t3 t4 </list> <except> {} </except> </allDifferent>
```

5.1.2.2 Constraint allEqual

Such constraints are similar to those described earlier for integer variables, but set variables simply replace integer variables.

```
Syntax 87
| <allEqual> <list> (setVar wspace)2+ </list> </allEqual>

Note that the opening and closing tags for `<list>` are optional, which gives:

```
Syntax 88
| <allEqual> (setVar wspace)2+ </allEqual>               // Simplified Form

Example 87
| <allEqual id="c"> s1 s2 s3 s4 </allEqual>
```

5.1.2.3 Constraint allIntersecting

The constraint `allIntersecting` ensures that each pair of set variables in `<list>` intersects in a number of elements that respects a numerical condition. It captures for example `at_most1` in MiniZinc, with operator `le` and value 1.
In the following example, the constraints \(c_1\) states that \(|s_1 \cap s_2| = k \land |s_1 \cap s_3| = k \land |s_2 \cap s_3| = k\), whereas the constraint \(c_2\) states that \(|t_1 \cap t_2| \leq 1 \land |t_1 \cap t_3| \leq 1 \land |t_2 \cap t_3| \leq 1\).

We propose to identify two cases of \texttt{allIntersecting} over set variables, corresponding to the case where all sets must be disjoint (value "disjoint" for \texttt{case}) and the case where all sets must be overlapping (value "overlapping" for \texttt{case}). For both cases, the element \texttt{<condition>} becomes implicit. The two following constraints \(c_3\) and \(c_4\):

\begin{verbatim}
<allIntersecting id="c3">
  <list> s1 s2 s3 </list>
  <condition> (gt,0) </condition>
</allIntersecting>
<allIntersecting id="c4">
  <list> t1 t2 t3 t4 </list>
  <condition> (eq,0) </condition>
</allIntersecting>
\end{verbatim}

can then be defined by:
5.1.2.4 Constraint ordered

As for the constraint ordered-set (i.e., the constraint ordered lifted to sets), we consider the inclusion order for sets: the operators that can be used are among \{\subseteq, \subseteq, \supseteq, \supset\}. This gives:

**Syntax 90**

```xml
<ordered>
  <list> (setVar wspace)2+ </list>
  <operator> "subset" | "subseq" | "supseq" | "supset" </operator>
</ordered>
```

**Semantics 53**

\[ \text{ordered}(S, \circ), \text{ with } S = \langle s_1, s_2, \ldots \rangle \text{ and } \circ \in \{\subseteq, \subseteq, \supseteq, \supset\}, \text{ iff} \]

\[ \forall i: 1 \leq i < |S|, s_i \circ s_{i+1} \]

This captures increasing (subsetEq in Choco3) and decreasing, as illustrated below respectively with constraints \(c_1\) and \(c_2\).

**Example 91**

```xml
<ordered id="c1">
  <list> s1 s2 s3 </list>
  <operator> subseq </operator>
</ordered>
<ordered id="c2">
  <list> t1 t2 t3 t4 t5 </list>
  <operator> supseq </operator>
</ordered>
```

A second form of ordered uses an underlying ordering based either on bounds or on indicator (characteristic) functions. In the former case, the ordering \(\preceq_{\text{bnd}}\) is defined as:

\[ s \preceq_{\text{bnd}} t \text{ iff } \max(s) \leq \min(t). \]

In the latter case, the ordering \(\preceq_{\text{ind}}\) is defined as follows:

\[ s \preceq_{\text{ind}} t \text{ iff } s = \emptyset \lor t \neq \emptyset \land (\min(s) < \min(t) \lor \min(s) = \min(t) \land s \setminus \{\min(s)\} \preceq_{\text{ind}} t \setminus \{\min(t)\}) \]

The attribute order is required here, with either the value "bnd" or the value "ind".
For the semantics, \texttt{ordered}^{\texttt{bnd}} and \texttt{ordered}^{\texttt{ind}} correspond to versions of \texttt{ordered}, with \texttt{order} set to "\texttt{bnd}" and "\texttt{ind}”, respectively.

The constraint with type "\texttt{bnd}" captures \texttt{sequence} of Gecode. The constraint \texttt{ordered} over set variables can be of course lifted to other structures like lists and sets. We show this on lists (sequences) of set variables, capturing \texttt{lex} defined in MiniZinc. The lexicographic order can rely on \texttt{\leq_{\texttt{bnd}}} or \texttt{\leq_{\texttt{ind}}}. For example, for \texttt{\leq_{\texttt{ind}}}, we have:

\[
S = \langle s_1, s_2, \ldots, s_k \rangle \leq_{\texttt{ind}} T = \langle t_1, t_2, \ldots, t_k \rangle \text{ iff } \\
\forall i: 1 \leq i < |S|, s_i \leq_{\texttt{ind}} s_{i+1} \\
S = \langle \rangle \lor T \neq \langle \rangle \land (s_1 \prec_{\texttt{ind}} t_1 \lor s_1 = t_1 \land \langle s_2, \ldots, s_k \rangle \leq_{\texttt{ind}} \langle t_2, \ldots, t_k \rangle).
\]

5.1.3 Counting Constraints

5.1.3.1 Constraint sum

The constraint \texttt{sum} ensures that the sum of coefficients in \texttt{<coeffs>} indexes by values present in \texttt{<index>} respects a numerical condition.
This constraint captures `sum_set` in the catalog, `sum` in Choco3 and `weights` in Gecode.

### 5.1.3.2 Constraint `count`

The constraint `count` ensures that the number of set variables in `<list>` which are assigned a set among those in `<values>` respects a numerical condition.

To simplify, we assume for the semantics that $T$ is a set of set values (and not variables).
Example 93

```xml
<count>
  <list> s1 s2 s3 s4 </list>
  <values> t </values>
  <condition> (eq,2) </condition>
</count>
```

The constraint count captures at Least, at Most and exactly. It also captures nEmpty of Choco3, by using <values> set() </values>.

5.1.3.3 Constraint range

The constraint range [16, 18] holds iff the set of values taken by variables of <list> at indices given by <index> is exactly the set <image>. The optional attribute startIndex gives the number used for indexing the first variable in <list> (0, by default).

Syntax 95

```xml
<range>
  <list [startIndex="integer"]> (intVar wspace)2+ </list>
  <index> setVal | setVar </index>
  <image> setVal | setVar </image>
</range>
```

Semantics 58

```
range(X,s,t), with X = \{x_1, x_2, \ldots\}, iff
{ x_i : 1 \leq i \leq |X| \land i \in s } = t
```

In the following example, the constraints c1 and c2, taken together, permit to represent nValues(\{x_0, x_1, x_2, x_3, x_4, x_5\}, y); see [16].

Example 94

```xml
<range id="c1">
  <list> x0 x1 x2 x3 x4 x5 </list>
  <index> \{0,1,2,3,4,5\} </index>
  <image> t </image>
</range>

<intension id="c2"> eq(y,card(t)) </intension>
```

5.1.3.4 Constraint roots

The constraint roots [16, 19] holds iff <index> represents exactly the set of indices of variables in <list> which are assigned a value in <image>. The optional attribute startIndex gives the number used for indexing the first variable in <list> (0, by default).
5.1.4 Connection Constraints

5.1.4.1 Constraint element

The constraint element ensures that <value> is element of <list>, i.e., equal to one set among those assigned to the set variables of <list>. The optional element <index> gives the index (position) of one occurrence of <value> inside <list>. The optional attribute startIndex gives the number used for indexing the first variable in <list> (0, by default).

Example 95

```
<roots id="c1">
  <list> x0 x1 x2 x3 x4 x5 </list>
  <index> s </index>
  <image> {0,1} </image>
</roots>

<intension id="c2"> eq(2,card(s)) </intension>
```
The first form (without element <index>) captures **member** in Choco3.

**Example 96**

```xml
<element id="c1">
  <list> s1 s2 s3 </list>
  <index> i </index>
  <value> t </value>
</element>

<element id="c2">
  <list> t1 t2 t3 t4 </list>
  <value> t5 </value>
</element>
```

5.1.4.2 **Constraint channel**

The constraint **channel**, in its first form, sometimes called **inverse set** or **inverse** in the literature, ensures that two lists of set variables represent inverse functions. Any value in the set of a list must be within the index range of the other list. For each list, the optional attribute **startIndex** gives the number used for indexing the first variable in this list (0, by default).

**Syntax 98**

```xml
<channel>
  <list [startIndex="integer"]> (setVar wspace)2+ </list>
  <list [startIndex="integer"]> (setVar wspace)2+ </list>
</channel>
```

For the semantics, starting indexes are assumed to be equal to 1.

**Semantics 61**

channel\((S, T)\), with \(S = \langle s_1, s_2, \ldots \rangle\) and \(T = \langle t_1, t_2, \ldots \rangle\), iff

\[
\forall i : 1 \leq i \leq |S|, j \in s_i \iff i \in t_j
\]

A second form of the constraint **channel**, called **int_set_channel** in MiniZinc, links a set of integer variables to a set of set variables.

**Syntax 99**

```xml
<channel>
  <list [startIndex="integer"]> (intVar wspace)+ </list>
  <list [startIndex="integer"]> (setVar wspace)+ </list>
</channel>
```

**Semantics 62**

channel\((X, S)\), with \(X = \langle x_1, x_2, \ldots \rangle\) and \(S = \langle s_1, s_2, \ldots \rangle\), iff

\[
\forall i : 1 \leq i \leq |X|, x_i = j \iff i \in s_j
\]
A final form, called \texttt{bool\_channel} in Choco3, is obtained by considering a list of 0/1 variables to be channeled with a set variable. It constrains this list to be a representation of the set.

### Syntax 100
\begin{verbatim}
<channel>
  <list [startIndex="integer"]> (01Var wspace)2+ </list>
  <value> setVar </value>
</channel>
\end{verbatim}

### Semantics 63
\begin{verbatim}
channel(X,s), with X = \{x_1,x_2,...\}, iff
\forall i: 1 \leq i \leq |X|, x_i = 1 \iff i \in s
\end{verbatim}

#### 5.1.4.3 Constraint partition
The constraint \texttt{partition} ensures that \texttt{<list>} represents a partition of \texttt{<value>}.

### Syntax 101
\begin{verbatim}
<partition>
  <list> (setVar wspace)2+ </list>
  <value> setVar </value>
</partition>
\end{verbatim}

### Semantics 64
\begin{verbatim}
partition(S,t), with S = \langle s_1,s_2,...\rangle, iff
| t = \bigcup_{i=1}^{|S|} s_i \land \forall (i,j): 1 \leq i < j \leq |S|, s_i \cap s_j = \emptyset
\end{verbatim}

#### 5.1.4.4 Constraint precedence
The constraint \texttt{precedence} ensures that if a set variable \( s_j \) of \texttt{<list>} contains the \( i+1 \)th value of \texttt{<values>}, but not the \( i \)th value, then another set variable of \texttt{<list>}, that precedes \( s_j \), contains the \( i \)th value of \texttt{<values>}, but not the \( i+1 \)th value. The optional attribute \texttt{covered} indicates whether each value of \texttt{<values>} must belong to at least one set variable of \texttt{<list>} ("false", by default). It is also possible to reverse the order by specifying the relational operator in the optional element \texttt{<operator>} (lt, by default).
For the semantics, $V^{cv}$ means that covered is "true", and we note $v \in S$ iff there exists a set variable $s_j \in S$ such that $v \in s_j$.

**Example 97**

```
<precedence>
  <list> s1 s2 s3 s4 </list>
  <values> 4 0 </values>
</precedence>
```

The constraint precedence captures set_value.precede in the Global Constraint Catalog.

### 5.2 Constraints over Graph Variables

In this section, we present constraints that are defined on graph variables. We have:

1. **circuit**
2. **nCircuits**
3. **path**
4. **nPaths**
5. **arbo**
6. **nArbos**
7. **nCliques**

#### 5.2.1 Constraint circuit

This form of the constraint circuit ensures that the value taken by the (directed or undirected) graph variable in `<graph>` represents a circuit (cycle). The optional element `<size>` indicates that the circuit must be of a given size (number of nodes). Consequently, in that case, and only in that case, the circuit is not required to cover all nodes that are present in the upper bound of the graph variable.
For the semantics, $n_{max}$ denotes the number of nodes in the upper bound $g_{max}$ of $g$.

### 5.2.2 Constraint nCircuits

For several circuits, one must specify a numerical condition.

#### Syntax 104

```
<nCircuits>
  <graph> graphVar </graph>
  <condition> "(" operator "," operand ")" </condition>
</nCircuits>
```

#### Semantics 67

\[
\text{nCircuits}(g, (\circ, k)), \text{ iff } g \text{ represents } p \text{ node-disjoint circuits with } p \circ k
\]

### 5.2.3 Constraint path

The constraint path ensures that the value taken by the (directed or undirected) graph variable in \(<\text{graph}>\) represents a path from the node specified in \(<\text{start}>\) to the node specified in \(<\text{final}>\). The optional element size indicates that the path must be of a given size (number of nodes). Consequently, in that case, and only in that case, the path is not required to cover all nodes that are present in the upper bound of the graph variable.

#### Syntax 105

```
<path>
  <graph> graphVar </graph>
  <start> intVal | intVar </start>
  <final> intVal | intVar </final>
  [<size> intVal | intVar </size>]
</path>
```
5.2.4 Constraint nPaths

For several paths, one must specify a numerical condition.

Syntax 106

```
<nPaths>
  <graph> graphVar </graph>
  <condition> "(" operator "," operand ")" </condition>
</nPaths>
```

Semantics 69

\[ \text{nPaths}(g,(\otimes,k)), \text{iff} \]
\[ g \text{ forms } p \text{ paths s.t. } p \otimes k \]

5.2.5 Constraint arbo

The constraint arbo ensures that the value taken by the directed (respectively, undirected) graph variable in \(<\text{graph}>\) represents an arborescence (respectively, tree) whose root is specified by \(<\text{root}>\). The optional element \text{size} indicates that the arborescence must be of a given size (number of nodes). Consequently, in that case, the arborescence is not required to cover all nodes that are present in the upper bound of the graph variable.

Syntax 107

```
<arbo>
  <graph> graphVar </graph>
  <root> intVal | intVar </root>
  [ <size> intVal | intVar </size> ]
</arbo>
```

Semantics 70

\[ \text{arbo}(g,r) \text{ iff } \]
\[ g \text{ is an arborescence of root } r \text{ and size } n_{\text{max}} \]
\[ \text{arbo}(g,r,s) \text{ iff } \]
\[ g \text{ is an arborescence of root } r \text{ and size } s > 1 \]
5.2.6 Constraint \texttt{nArbos}

Syntax 108

\[
\begin{align*}
\texttt{nArbos} & \quad <\texttt{graph}> \texttt{graphVar} <\texttt{/graph}> \\
& \quad <\texttt{condition}> "(\ " \texttt{operator} \ " , \ " \texttt{operand} \ " )" <\texttt{/condition}> \\
& \quad \texttt{/nArbos}> 
\end{align*}
\]

Semantics 71

\[
\begin{align*}
\texttt{nArbos} & (g, (\odot, k)) \text{, with } \odot \in \{<, \leq,\geq,=,\neq\}, \text{ iff } \\
g & \text{ represents } p \text{ arborescences s.t. } p \odot k
\end{align*}
\]

5.2.7 Constraint \texttt{nCliques}

The constraint \texttt{nCliques}, called \texttt{nclique} in [47], ensures that the value taken by the graph variable in \texttt{<graph>} represents a set of cliques whose cardinality must respect a numerical condition.

Syntax 109

\[
\begin{align*}
\texttt{nCliques} & \quad <\texttt{graph}> \texttt{graphVar} <\texttt{/graph}> \\
& \quad <\texttt{condition}> "(\ " \texttt{operator} \ " , \ " \texttt{operand} \ " )" <\texttt{/condition}> \\
& \quad \texttt{/nCliques}> 
\end{align*}
\]

Semantics 72

\[
\begin{align*}
\texttt{nCliques} & (g, (\odot, k)) \text{, with } \odot \in \{<, \leq,\geq,=,\neq\}, \text{ iff } \\
g & \text{ represents } p \text{ cliques s.t. } p \odot k
\end{align*}
\]
In this chapter, we introduce constraints over continuous variables, namely, real and qualitative variables. We shall use the term realVar and qualVar to denote a real variable and a qualitative variable, respectively. These terms as well as related ones, like for example, realVal, are defined in Appendix B.

6.1 Constraints over Real Variables

Reasoning over real variables is typically performed by means of intensional arithmetic constraints and sum (linear) constraints. However, in the future, we might introduce a few global constraints adapted to real variables.

6.1.1 Constraint intension

A constraint intension on real variables is built similarly to a constraint intension defined on integer variables. Simply, some additional operators are available. They are given by Table 6.1 that does show the specific operators that can be used to build predicates with real operands. Of course, it is possible to combine such operators with those presented for integers (and Booleans) in Table 4.1.

An intensional constraint is defined by an element <intension>, containing an element <function> that describes the functional representation of the predicate, referred to as bool-ExprReal in the syntax box above, and whose precise syntax is given in Appendix B.
Note that the opening and closing tags for `<function>` are optional, which gives:

**Syntax 111**

```xml
<intension boolExprReal>
</intension>  // Simplified Form
```

<table>
<thead>
<tr>
<th>Operation</th>
<th>Arity</th>
<th>Syntax</th>
<th>Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constants</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pi</td>
<td>0</td>
<td>PI</td>
<td>π</td>
</tr>
<tr>
<td>Euler’s constant</td>
<td>0</td>
<td>E</td>
<td>e</td>
</tr>
<tr>
<td><strong>Arithmetic</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real Division</td>
<td>2</td>
<td>fdiv(x, y)</td>
<td>x/y</td>
</tr>
<tr>
<td>Real Remainder</td>
<td>2</td>
<td>fmod(x, y)</td>
<td>x%y</td>
</tr>
<tr>
<td><strong>Functional</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Square root</td>
<td>1</td>
<td>sqrt(x)</td>
<td>√x</td>
</tr>
<tr>
<td>nth root</td>
<td>2</td>
<td>nroot(x, n)</td>
<td>√[^n]x</td>
</tr>
<tr>
<td>Natural exponential</td>
<td>1</td>
<td>exp(x)</td>
<td>e^x</td>
</tr>
<tr>
<td>Natural Logarithm</td>
<td>1</td>
<td>ln(x)</td>
<td>log_e x</td>
</tr>
<tr>
<td>Logarithm to base</td>
<td>2</td>
<td>log(x, b)</td>
<td>log_b x</td>
</tr>
<tr>
<td>Sine</td>
<td>1</td>
<td>sin(x)</td>
<td>sin x</td>
</tr>
<tr>
<td>Cosine</td>
<td>1</td>
<td>cos(x)</td>
<td>cos x</td>
</tr>
<tr>
<td>Tangent</td>
<td>1</td>
<td>tan(x)</td>
<td>tan x</td>
</tr>
<tr>
<td>Arcsine</td>
<td>1</td>
<td>asin(x)</td>
<td>arcsin x</td>
</tr>
<tr>
<td>Arcosine</td>
<td>1</td>
<td>acos(x)</td>
<td>arccos x</td>
</tr>
<tr>
<td>Arctangent</td>
<td>1</td>
<td>atan(x)</td>
<td>arctan x</td>
</tr>
<tr>
<td>Hyperbolic sine</td>
<td>1</td>
<td>sinh(x)</td>
<td>sinh x</td>
</tr>
<tr>
<td>Hyperbolic cosine</td>
<td>1</td>
<td>cosh(x)</td>
<td>cosh x</td>
</tr>
<tr>
<td>Hyperbolic tangent</td>
<td>1</td>
<td>tanh(x)</td>
<td>tanh x</td>
</tr>
</tbody>
</table>

Table 6.1: Operators on reals that can be used to build predicates.

In the following example, the constraints $c_1$ and $c_2$ are respectively defined by $v = \sqrt{w}$ and $x \geq 3 \ln y + z^4$, with $v, w, x, y, z$ assumed to be real variables.

**Example 98**

```xml
<intension id="c1">
  eq(v, sqrt(w))
</intension>

<intension id="c2">
  ge(x, add(mul(3, ln(y)), pow(z, 4)))
</intension>
```
6.1.2 Constraint sum

The constraint sum is the most important constraint in the context of real computation. When the optional element <coeffs> is missing, it is assumed that all coefficients are equal to 1. Of course, real values, variables and intervals are expected in <condition>.

Syntax 112

```
<sum>
  <list> (realVar wspace)2+ </list>
  [coeffs> (realVal wspace)+ </coeffs>
  <condition> "(" operator "," (realVal | realVar | realIntvl) ")" </condition>
</sum>
```

Semantics 73

\[ \text{sum}(X, C, (\odot, k)), \text{ with } X = \langle x_1, x_2, \ldots \rangle \text{ and } C = \langle c_1, c_2, \ldots \rangle, \text{ iff} \]
\[ \left( \sum_{i=1}^{\mid X \mid} c_i \times x_i \right) \odot k \]

Prerequisite: \( \mid X \mid = \mid C \mid \)

In the following example, the constraint expresses \( 1.5x_0 + 2x_1 + 3.2x_2 > y \).

Example 99

```
<sum>
  <list> x0 x1 x2 </list>
  <coeffs> 1.5 2 3.2 </coeffs>
  <condition> (gt, y) </condition>
</sum>
```

6.2 Constraints over Qualitative Variables

Qualitative Spatial Temporal Reasoning (QSTR) deals with qualitative calculi (also called algebras). A qualitative calculus is defined from a finite set \( B \) of base relations on a certain domain. In this section, we introduce some constraints over some important qualitative calculi. In the future, more algebras might be introduced.

6.2.1 Constraint interval

Interval Algebra, also called Allen’s calculus [2] handles temporal entities that represent intervals on the rational line. The set of base relations of this calculus is:

\[ B_{int} = \{eq, p, pi, m, mi, o, oi, s, si, d, di, f, fi\} \]

where for example \( m \) stands for meets.

The constraint interval allows us to express qualitative information between two intervals.
In the following example, the constraint expresses that either $x$ must overlap $y$ or $x$ must meet $y$.

Example 100

\[
\langle \text{interval} \rangle \\
\langle \text{scope} \rangle x \ y \ \rangle \\
\langle \text{relation} \rangle o \ m \ \rangle \\
\langle /\text{interval} \rangle
\]

6.2.2 Constraint point

Point Algebra (PA) is a simple qualitative algebra defined for time points. The set of base relations of this calculus is:

\[ B_{\text{point}} = \{ b, eq, a \} \]

where for example $b$ stands for before.

The constraint point allows us to express qualitative information between two points.

In the following example, the constraint expresses that either $x$ must be before or after $y$.
6.2.3 Constraint rcc8

RCC8 [110] is a region connection calculus for reasoning about regions in Euclidean space. The set of base relations of this calculus is:

\[ B_{rcc8} = \{dc, ec, eq, po, tpp, tppi, nttp, ntppi\} \]

where for example \(dc\) stands for disconnected.

The constraint \(\text{region}\) allows us to express qualitative information between two regions. For RCC8, the attribute \(\text{type}\) is required, and its value must be set to "rcc8".

### Syntax 115

\[
\begin{align*}
\text{region} & \quad \text{type}="rcc8" \\
\text{scope} & \quad \text{qualVar} \; \text{wspace} \; \text{qualVar} \\
\text{relation} & \quad (rcc8\text{BaseRelation} \; \text{wspace})*
\end{align*}
\]

### Semantics 76

\[
\text{region}^{rcc8}(x, y, R), \text{ with } R \subseteq B_{rcc8}, \text{ iff} \\
\exists b \in R : (x, y) \in b
\]

In the following example, the constraint expresses that either \(x\) must be disconnected or externally connected with \(y\).

### Example 102

\[
\begin{align*}
\text{region} & \quad \text{type}="rcc8" \\
\text{scope} & \quad x \; y \\
\text{relation} & \quad \text{dc} \; \text{ec}
\end{align*}
\]

6.2.4 Constraint dbd

For Temporal Constraint Satisfaction [43], variables represent time points and temporal information is represented by a set of unary and binary constraints, each specifying a set of permitted intervals.

For this framework, we only need to introduce a type of constraints, called \(\text{dbd}\) constraints, for disjunctive binary difference constraints (as in [72]).
Syntax 116

\[
\langle \text{dbd} \rangle \\
\langle \text{scope} \rangle \text{qualVar (wspace qualVar)* } \langle /\text{scope} \rangle \\
\langle \text{intervals} \rangle \text{ (realIntvl wspace)* } \langle /\text{intervals} \rangle \\
\langle /\text{dbd} \rangle
\]

Below, we give the semantics for unary and binary \text{dbd} constraints.

Semantics 77

\[
\text{dbd}(x, I), \text{ with } I = (l_1..u_1, l_2..u_2, ...), \text{ iff } \\
\exists i : 1 \leq i \leq |I| \land x \in l_i..u_i
\]

\[
\text{dbd}(x, y, I), \text{ with } I = (l_1..u_1, l_2..u_2, ...), \text{ iff } \\
\exists i : 1 \leq i \leq |I| \land y - x \in l_i..u_i
\]

In the following example, the constraint expresses that either \(x_2 - x_1\) must be in \([30, 40]\) or in \([60, +\text{infinity}]\).

Example 103

\[
\langle \text{dbd} \rangle \\
\langle \text{scope} \rangle x_1 x_2 \langle /\text{scope} \rangle \\
\langle \text{intervals} \rangle [30, 40] [60, +\text{infinity}] \langle /\text{intervals} \rangle \\
\langle /\text{dbd} \rangle
\]
Part III

Advanced Forms of Constraints
In XCSP3, it is possible to build advanced forms of (basic) constraints by means of lifting, restriction, sliding, logical combination and relaxation mechanisms. In this part, we introduce them.
In this chapter, we introduce two mechanisms that allow us to extend the scope of constraints defined in XCSP3. First, we show how it is possible to lift constraints over lists, sets, multisets, and also matrices, in a quite generic and natural way. Second, we show how classical restrictions on constraints can be posed by operating a simple attribute. These two mechanisms offer flexibility and extensibility, without paying the price of introducing many new XML elements.

7.1 Constraints lifted to Lists, Sets and Multisets

Many constraints, introduced earlier on integer variables can be extended to lists (tuples), sets and multisets. In XCSP3, this is quite easy to handle: replace, when appropriate, each integer variable of a list by an element `<list>`, or replace each of them by an element `<set>`, or even replace each of them by an element `<mset>`. The semantics, initially given for a sequence of variables, is naturally extended to apply to a sequence of lists of variables, a sequence of sets of variables, and a sequence of multisets of variables. The semantics must handle now tuples of values, sets of values, and multisets of values, since:

- The values assigned to the variables of an element `<list>` represent a tuple of values. For example, if we have `<list> x1 x2 x3 </list>` and the instantiation $x_1 = 1, x_2 = 0, x_3 = 1$, we deal with the tuple $(1, 0, 1)$.

- The values assigned to the variables of an element `<set>` represent a set of values. For example, if we have `<set> x1 x2 x3 </set>` and the instantiation $x_1 = 1, x_2 = 0, x_3 = 1$, we deal with the set $\{0, 1\}$.

- The values assigned to the variables of an element `<mset>` represent a multiset of values. For example, if we have `<mset> x1 x2 x3 </mset>` and the instantiation $x_1 = 1, x_2 = 0, x_3 = 1$, we deal with the multiset $\{0, 1, 1\}$.
A lifting operation always applies to an element `<list>` conceived to contain integer variables. After all variables have been replaced by lists, sets or tuples, the opening and closing tags for the initial element `<list>` are no more required. This is the reason we shall never represent them.

In this section, we show this approach for the most popular “basic” constraints. More specifically, we present the following constraints lifted to lists, sets and multisets:

- `allDifferent`
- `allEqual`
- `allDistant`
- `ordered`
- `allIncomparable`
- `nValues`
- `sort`

Because these constraints are defined on several sequences (vectors) of variables, they admit a parameter $X = \langle X_1, X_2, \ldots \rangle$, with $X_1 = \langle x_{1,1}, x_{1,2}, \ldots \rangle$, $X_2 = \langle x_{2,1}, x_{2,2}, \ldots \rangle$, etc. If $X = \langle x_1, x_2, \ldots, x_p \rangle$ then $|X| = p$, and:

- $X$ denotes $\langle x_1, x_2, \ldots, x_p \rangle$, the tuple of values obtained when considering an arbitrary instantiation of $X$.
- $\{X\}$ denotes $\{x_i : 1 \leq i \leq p\}$, the set of values obtained when considering an arbitrary instantiation of $X$.
- $\{\{X\}\}$ denotes $\{\{x_i : 1 \leq i \leq p\}\}$, the multiset of values obtained when considering an arbitrary instantiation of $X$.

Although we do not introduce new constraint types (names) in XCSP3, in order to clarify the textual description (in particular, for the semantics), we shall refer in the text to the versions of a constraint $\mathit{ctr}$, lifted to lists, sets and multisets, by $\mathit{ctr-list}$, $\mathit{ctr-set}$ and $\mathit{ctr-mset}$, respectively. For example, we shall refer in the text to $\mathit{allDifferent-list}$, $\mathit{allDifferent-set}$ and $\mathit{allDifferent-mset}$ when considering lifted versions of $\mathit{allDifferent}$.

### 7.1.1 Lifted Constraints `allDifferent`

The constraint `allDifferent`, introduced earlier on integer variables, can be naturally extended to lists (tuples), sets and multisets [107].

#### 7.1.1.1 On lists (tuples)

If `allDifferent` admits as parameter several lists of integer variables, then the constraint ensures that the tuple of values taken by variables of the first element `<list>` is different from the tuple of values taken by variables of the second element `<list>`. If more than two elements `<list>` are given, all tuples must be different. A variant enforces tuples to take distinct values, except those that are assigned to some specified tuples (often, the single tuple containing only 0), specified in the optional element `<except>`.
As explained in the introduction of this section, in the text below, allDifferent-list refers to allDifferent defined over several lists of integer variables.

**Semantics**

\( \text{allDifferent-list}(\mathcal{X}, E) \), with \( \mathcal{X} = \langle X_1, X_2, \ldots \rangle \), \( E \) the set of discarded tuples, iff

\[
\forall (i,j) : 1 \leq i < j \leq |\mathcal{X}|, X_i \neq X_j \lor X_i \in E \lor X_j \in E
\]

allDifferent-list(\( \mathcal{X} \)) iff allDifferent-list(\( \mathcal{X}, \emptyset \))

**Prerequisite:** \( |\mathcal{X}| \geq 2 \land \forall i : 1 \leq i < |\mathcal{X}|, |X_i| = |X_{i+1}| \geq 2 \land \forall \tau \in E, |\tau| = |X_1| \)

**Example 104**

\[
\begin{align*}
\text{allDifferent id="c1"} & \quad \text{<list> x1 x2 x3 x4 </list>} \\
\text{allDifferent id="c2"} & \quad \text{<list> v1 v2 v3 v4 </list>}
\end{align*}
\]

Constraints captured by allDifferent-list from the Global Constraint Catalog:

- lex_different
- lex_alldifferent
- lex_alldifferent_except_0

### 7.1.1.2 On sets

If allDifferent admits several sets of integer variables as parameter, then the constraint ensures that the set of values taken by variables of the first element <set> is different from the set of values taken by variables of the second element <set>. If more than two elements <set> are given, all sets must be different. A variant enforces sets of values to be different, except those that are assigned to some specified sets, specified in the optional element <except>. 
As explained in the introduction of this section, in the text below, `allDifferent-set` refers to `allDifferent` defined over several sets of integer variables.

**Semantics 79**

allDifferent-set($X, E$), with $X = \{X_1, X_2, \ldots\}$, $E$ the set of discarded sets, iff

\[ \forall (i, j) : 1 \leq i < j \leq |X|, \{X_i\} \neq \{X_j\} \lor \{X_i\} \in E \lor \{X_j\} \in E \]

allDifferent-set($X$) iff allDifferent-set($X, \emptyset$)

**Prerequisite:** $|X| \geq 2 \land \forall i : 1 \leq i \leq |X|, |X_i| \geq 1$

**Example 105**

```
<allDifferent id="c1">
  <set> x1 x2 x3 x4 </set>
  <set> y1 y2 y3 </set>
</allDifferent>

<allDifferent id="c2">
  <set> v1 v2 v3 </set>
  <set> w1 w2 w3 w4 </set>
  <set> z1 z2 </set>
  <except> {0,1} </except>
</allDifferent>
```

### 7.1.1.3 On multisets

If `allDifferent` admits several multisets of integer variables as parameter, then the constraint ensures that the multiset of values taken by variables of the first element `<mset>` is different from the multiset of values taken by variables of the second element `<mset>`. If more than two elements `<mset>` are given, all multisets must be different. A variant enforces multisets of values to be different, except those that correspond some specified multisets, specified in the optional element `<except>`.

**Syntax 119**

```
<allDifferent>
  (<mset> (intVar wspace)+ </mset>)2+
  [<except> ("{" intVal ("," intVal)+ "}")]+ </except>
</allDifferent>
```

As explained in the introduction of this section, in the text below, `allDifferent-mset` refers to `allDifferent` defined over several multisets of integer variables.
Semantics 80

\( \text{allDifferent-mset}(X, E) \), with \( X = \langle X_1, X_2, \ldots \rangle \), \( E \) the set of discarded msets, iff
\[
\forall (i, j) : 1 \leq i < j \leq |X|, \{\{X_i\}\} \neq \{\{X_j\}\} \lor \{\{X_i\}\} \in E \lor \{\{X_j\}\} \in E
\]
\( \text{allDifferent-mset}(X) \) iff \( \text{allDifferent-mset}(X, \emptyset) \)

Prerequisite: \( |X| \geq 2 \land \forall i : 1 \leq i < |X|, |X_i| = |X_{i+1}| \geq 2 \land \forall M \in E, |M| = |X_1| \)

Example 106

```xml
<allDifferent id="c1">
  <mset> x1 x2 x3 x4 </mset>
  <mset> y1 y2 y3 y4 </mset>
</allDifferent>

<allDifferent id="c2">
  <mset> v1 v2 v3 v4 </mset>
  <mset> w1 w2 w3 w4 </mset>
  <mset> z1 z2 z3 z4 </mset>
  <except> {{0,0,0,0}} </except>
</allDifferent>
```

7.1.2 Lifted Constraints allEqual

The versions of allEqual lifted to lists, sets and multisets are defined similarly to those presented above for allDifferent.

7.1.2.1 On lists (tuples)

Syntax 120

```xml
<allEqual>
  (<list> (<intVar wspace>)2+ </list>)2+
</allEqual>
```

Semantics 81

\( \text{allEqual-list}(X) \), with \( X = \langle X_1, X_2, \ldots \rangle \), iff
\[
\forall (i, j) : 1 \leq i < j \leq |X|, X_i = X_j
\]

Prerequisite: \( |X| \geq 2 \land \forall i : 1 \leq i < |X|, |X_i| = |X_{i+1}| \geq 2 \)

Constraints captured by allEqual-list from the Global Constraint Catalog: lex_equal
7.1.2.2 On sets

Syntax 121

```xml
<allEqual>
  (<set> (intVar wspace)+ </set>)2+
</allEqual>
```

Semantics 82

\text{allEqual-set}(\mathcal{X}), \text{ with } \mathcal{X} = \langle X_1, X_2, \ldots \rangle, \text{ iff}
\forall(i,j) : 1 \leq i < j \leq |\mathcal{X}|, \{X_i\} = \{X_j\}

\text{Prerequisite} : |\mathcal{X}| \geq 2 \land \forall i : 1 \leq i \leq |\mathcal{X}|, |X_i| \geq 1

7.1.2.3 On multisets

Syntax 122

```xml
<allEqual>
  (<mset> (intVar wspace)+ </mset>)2+
</allEqual>
```

Semantics 83

\text{allEqual-mset}(\mathcal{X}), \text{ with } \mathcal{X} = \langle X_1, X_2, \ldots \rangle, \text{ iff}
\forall(i,j) : 1 \leq i < j \leq |\mathcal{X}|, \{X_i\} = \{X_j\}

\text{Prerequisite} : |\mathcal{X}| \geq 2 \land \forall i : 1 \leq i \leq |\mathcal{X}|, |X_i| = |X_{i+1}| \geq 2

Constraints captured by \text{allEqual-mset} from the Global Constraint Catalog: same

7.1.3 Lifted Constraints allDistant

Similarly to \textit{allDifferent} and \textit{allEqual}, \textit{allDistant} can be lifted to lists, sets and multisets. Of course, we have to clarify the distances that are used, when considering pairs of lists, sets and multisets. By default, we shall use the following distances:

- Hamming distance for lists: \( dist_H(X,Y) = |\{i : 1 \leq i \leq |X| \land x_i \neq y_i\}| \) with \( X = \langle x_1, x_2, \ldots \rangle, Y = \langle y_1, y_2, \ldots \rangle \) and \( |X| = |Y| \).
- \textit{infimum distance} for sets (of integers): \( dist_{\inf}(X,Y) = \min\{ |x - y| : x \in X \land y \in Y \} \)
- \textit{Manhattan distance} for multisets: \( dist_M(X,Y) = \sum_{a \in X \cup Y} |\nu_X(a) - \nu_Y(a)| \) with \( \nu_Z(c) \) denoting the multiplicity (number of occurrences) of value \( c \) in the multiset \( Z \).
7.1.3.1 On lists (tuples)

Syntax 123

```xml
<allDistant>
  (<list> (intVar wspace)2+ </list>)+
  <condition> "(" operator "," operand ")" </condition>
</allDistant>
```

Semantics 84

```
allDistant-list(\mathcal{X},(\odot,k)),\text{ with }\mathcal{X} = (X_1,X_2,...), \text{ iff } \\
\forall(i,j):1 \leq i < j \leq |\mathcal{X}|, dist_H(X_i,X_j) \odot k
```

Prerequisite: \(|\mathcal{X}| \geq 2 \land \forall i : 1 \leq i \leq |\mathcal{X}|, |X_i| = |X_{i+1}| \geq 2\)

Constraints captured by allDistant-list from the Global Constraint Catalog:

- differ_from_at_least_k_pos, differ_from_at_most_k_pos
- differ_from_exactly_k_pos
- all_differ_from_at_least_k_pos, all_differ_from_at_most_k_pos
- all_differ_from_exactly_k_pos

7.1.3.2 On sets

Syntax 124

```xml
<allDistant>
  (<set> (intVar wspace)+ </set>)+
  <condition> "(" operator "," operand ")" </condition>
</allDistant>
```

Semantics 85

```
allDistant-set(\mathcal{X},(\odot,k)),\text{ with }\mathcal{X} = (X_1,X_2,...), \text{ iff } \\
\forall(i,j):1 \leq i < j \leq |\mathcal{X}|, dist_{\mathcal{I}}(\{X_i\},\{X_j\}) \odot k
```

Prerequisite: \(|\mathcal{X}| \geq 2 \land \forall i : 1 \leq i \leq |\mathcal{X}|, |X_i| \geq 1\)

We propose to identify two types of allDistant-set, corresponding to the case where all sets must be disjoint (value "disjoint" for case) and the case where all sets must be overlapping (value "overlapping" for case). For both cases, the element <condition> becomes implicit. The two following constraints \(c_1\) and \(c_2\):
can then be defined by:

### 7.1.3.3 On multisets

#### Syntax 125

```xml
<allDistant>
  (<mset> ((intVar wspace)2+ </mset>)2+ <condition> "(" operator "," operand ")" </condition>
</allDistant>
```

#### Semantics 86

Given a multiset \( \mathcal{X} = \langle X_1, X_2, \ldots \rangle \), the \( k \)-th power of \( \mathcal{X} \) is defined as

\[
\mathcal{X}^k = (\mathcal{X} \odot k)^k = (\mathcal{X} \odot k)^{k^2}\]

**Prerequisite**: \( |\mathcal{X}| \geq 2 \) and \( 1 \leq i < |\mathcal{X}|, |X_i| = |X_{i+1}| = 2 \)

**Remark 31** In the future, we project to deal with other distances by introducing an attribute distance, while fixing the terminology (values that can be used for distance).
7.1.4 Lifted Constraints ordered (lex on lists)

The constraint ordered can be naturally lifted to lists, sets and multisets. As for allDistant, we have to clarify the (total or partial) orders that are used, when comparing lists, sets and multisets. By default, we shall use the following orders:

- lexicographic order \( \leq_{lex} \) for lists;
- inclusion order \( \subseteq \) for sets;
- multiplicity inclusion order \( \subseteq \) for multisets: \( X \subseteq Y \) iff for all \( a \in X, \nu_X(a) \leq \nu_Y(a) \), with \( \nu_Z(c) \) denoting the multiplicity (number of occurrences) of value \( c \) in the multiset \( Z \).

7.1.4.1 On lists (tuples)

Because this constraint is very popular, it is allowed to use lex, instead of ordered over lists of integer variables. The constraint lex, see [32, 56], ensures that the tuple formed by the values assigned to the variables of the first element \( <\text{list}> \) is related to the tuple formed by the values assigned to the variables of the second element \( <\text{list}> \) with respect to the operator specified in \( <\text{operator}> \). If more than two elements \( <\text{list}> \) are given, the entire sequence of tuples must be ordered; this captures then lexChain [31].

**Syntax 126**

```xml
<lex>
  (<list> (intVar wspace)2+ <list>)2+
  <operator> "lt" | "le" | "ge" | "gt" </operator>
</lex>
```

**Semantics 87**

\[ \text{lex}(X, \odot), \text{with } X = \langle X_1, X_2, \ldots \rangle \text{ and } \odot \in \{\lt, \leq, \geq, \gt\}, \text{ iff} \]
\[ \forall i : 1 \leq i < |X|, X_i \odot X_{i+1} \]

**Prerequisite**: \( |X| \geq 2 \land \forall i : 1 \leq i < |X|, |X_i| = |X_{i+1}| \geq 2 \)

In the following example, the constraint \( c_1 \) states that \( \langle x_1, x_2, x_3, x_4 \rangle \leq_{lex} \langle y_1, y_2, y_3, y_4 \rangle \), whereas \( c_2 \) states that \( \langle z_1, z_2, z_3 \rangle \geq_{lex} \langle z_4, z_5, z_6 \rangle \geq_{lex} \langle z_7, z_8, z_9 \rangle \).

**Example 109**

```xml
<lex id="c1">
  <list> x1 x2 x3 x4 </list>
  <list> y1 y2 y3 y4 </list>
  <operator> le </operator>
</lex>

<lex id="c2">
  <list> z1 z2 z3 </list>
  <list> z4 z5 z6 </list>
  <list> z7 z8 z9 </list>
  <operator> gt </operator>
</lex>
```
Constraints captured by lex from the Global Constraint Catalog:
- lex_between
- lex_greater, lex_greatereq
- lex_less, lex_lesseq
- lex_chain_greater, lex_chain_greatereq
- lex_chain_less, lex_chain_lesseq

7.1.4.2 On sets

The constraint ensures that the set of values taken by variables of the first element <set> is related to the set of values taken by the variables of the second element <set>, with respect to a relational set operator. If more than two elements <set> are given, the entire sequence of sets must be ordered.

**Syntax 127**

```
<ordered>
  (<set> (intVar wspace)+ </set>)2+
  <operator> "subset" | "subseq" | "supseq" | "supset" </operator>
</ordered>
```

**Semantics 88**

ordered-set(\(\mathcal{X}, \odot\)), with \(\mathcal{X} = \{X_1, X_2, \ldots\}\) and \(\odot \in \{\subseteq, \subseteq, \supseteq, \supseteq\}\), iff
\[
\forall i : 1 \leq i < |\mathcal{X}|, \{X_i\} \odot \{X_{i+1}\}
\]

Prerequisite: \(|\mathcal{X}| \geq 2 \land \forall i : 1 \leq i < |\mathcal{X}|, |X_i| \geq 1\)

This captures uses [16].

7.1.4.3 On multisets

**Syntax 128**

```
<ordered>
  (<mset> (intVar wspace)+ </mset>)2+
  <operator> "subset" | "subseq" | "supseq" | "supset" </operator>
</ordered>
```

**Semantics 89**

ordered-mset(\(\mathcal{X}, \odot\)), with \(\mathcal{X} = \{X_1, X_2, \ldots\}\) and \(\odot \in \{\subseteq, \subseteq, \supseteq, \supseteq\}\), iff
\[
\forall i : 1 \leq i < |\mathcal{X}|, \{|X_i|\} \odot \{|X_{i+1}|\}
\]

Prerequisite: \(|\mathcal{X}| \geq 2 \land \forall i : 1 \leq i < |\mathcal{X}|, |X_i| = |X_{i+1}| \geq 2\)

This captures usedBy [11].
Remark 32 In the future, we project to deal with other orders by introducing an attribute order, while fixing the terminology (values that can be used for order).

7.1.5 Lifted Constraints allIncomparable

The constraint allIncomparable is not defined on integer variables because \( \mathbb{N} \) is totally ordered, but it is meaningful on lists, sets and multisets. This is why it is only introduced in this chapter. As for ordered, we have to clarify the partial orders that are used, when comparing lists, sets and multisets. By default, we shall use the following partial orders:

- product order (and not lexicographic order) for lists: \( X \leq_{\text{prod}} Y \) iff \( \forall i : 1 \leq i \leq |X|, x_i \leq y_i \) with \( X = (x_1, x_2, \ldots) \), \( Y = (y_1, y_2, \ldots) \) and \( |X| = |Y| \);
- inclusion order \( \subseteq \) for sets;
- multiplicity inclusion order \( \subseteq \) for multisets: \( X \subseteq Y \) iff \( \forall a \in X, \nu_X(a) \leq \nu_Y(a) \), with \( \nu_Z(c) \) denoting the multiplicity (number of occurrences) of value \( c \) in the multiset \( Z \).

7.1.5.1 On lists (tuples)

All elements \(<list>\) must correspond to tuples that are all incomparable (for the product order).

![Syntax 129](image)

Semantics 90

\[
\text{allIncomparable-list}(\mathcal{X}), \text{ with } \mathcal{X} = (X_1, X_2, \ldots) \text{, iff } \forall (i, j) : 1 \leq i < j \leq |\mathcal{X}|, \exists (k, l) : 1 \leq k < l \leq |X_i| \land x_{i,k} > x_{j,k} \land x_{i,l} < x_{j,l}
\]

Prerequisite: \( |\mathcal{X}| \geq 2 \land \forall i : 1 \leq i < |\mathcal{X}|, |X_i| = |X_{i+1}| \geq 2 \)

The constraints captured by allIncomparable-list from the Global Constraint Catalog are: incomparable and all_incomparable

7.1.5.2 On sets

All elements \(<set>\) must be incomparable (for inclusion).

![Syntax 130](image)
7.1.5.3 On multisets

All elements <mset> must be incomparable (for multiplicity inclusion).

Syntax 131

```
<allIncomparable>
  (<mset> (<intVar> wspace)+ </mset>)2+ </allIncomparable>
```

Semantics 92

allIncomparable-mset(\(\mathcal{X}\)), with \(\mathcal{X} = (X_1, X_2, \ldots)\), iff
\[
\forall (i, j) : 1 \leq i < j \leq |\mathcal{X}|, \{\{X_i\}\} \not\subseteq \{\{X_j\}\} \land \{\{X_j\}\} \not\subseteq \{\{X_i\}\}
\]

Prerequisite: \(|\mathcal{X}| \geq 2 \land \forall i : 1 \leq i \leq |\mathcal{X}|, |X_i| \geq 1\)

Remark 33 In the future, we project to deal with other partial orders by introducing an attribute order, while fixing the terminology (values that can be used for order).

7.1.6 Lifted Constraints nValues

The constraint nValues can be naturally lifted to lists, sets and multisets; the term “value” being understood successively as a list, a set or a multiset.

7.1.6.1 On lists

This constraint, sometimes called nVectors, ensures that the number of distinct tuples taken by variables of different elements <list> (of same size) must respect a numerical condition. A variant enforces tuples to take distinct values, except those that are assigned to some specified tuples (often, the single tuple containing only 0), specified in the optional element <except>.

Syntax 132

```
<nValues>
  (<list> (<intVar> wspace)+ </list>)2+ <except> ("(" <intVal> "," <intVal>+ ")")+ </except>
  <condition> "(" <operator> "," <operand> ")" </condition>
</nValues>
```

For the semantics, \(E\) denotes the set of tuples that must be discarded.
Semantics 93
nValues-list(\(X, E, (\odot, k)\)), with \(X = \langle X_1, X_2, \ldots \rangle\), iff
\[|\{X_i : 1 \leq i \leq |X|\} \setminus E| \odot k\]
nValues(\(X, (\odot, k)\)) iff nValues(\(X, \emptyset, (\odot, k)\))

\textbf{Prerequisite}: \(|X| \geq 2 \land \forall 1 \leq i < |X|, |X_i| = |X_{i+1}| \geq 2 \land \forall \tau \in E, |\tau| = |X_i|\)

Example 110

\[
\begin{align*}
\text{<nValues id="c">} \\
\text{<list> } & x1 \ x2 \ x3 \ <\/list> \\
\text{<list> } & y1 \ y2 \ y3 \ <\/list> \\
\text{<list> } & z1 \ z2 \ z3 \ <\/list> \\
\text{<condition> } & (eq, 2) \ <\/condition> \\
\text{</nValues>}
\end{align*}
\]

Constraints captured by nValues-list from the Global Constraint Catalog:
- atleast_nvector, atmost_nvector
- nvector, nvectors

7.1.6.2 On sets

Syntax 133

\[
\begin{align*}
\text{<nValues>} \\
& \hspace{1em} \langle \text{set} \rangle \ (\text{intVar } \text{wspace})* \langle \text{set} \rangle \rangle 2+ \\
& \hspace{1em} \langle \text{except} \rangle \ ("\{ \text{intVal} \ (", \ \text{intVal}) \}")+ \langle \text{except} \rangle \\
\text{<condition> } & (" \text{operator } ", \ \text{operand } ") \ <\/condition> \\
\text{</nValues>}
\end{align*}
\]

Semantics 94
nValues-set(\(X, E, (\odot, k)\)), with \(X = \langle X_1, X_2, \ldots \rangle\), iff
\[|\{\{X_i\} : 1 \leq i \leq |X|\} \setminus E| \odot k\]
nValues-set(\(X, (\odot, k)\)) iff nValues(\(X, \emptyset, (\odot, k)\))

\textbf{Prerequisite}: \(|X| \geq 2 \land \forall 1 \leq i \leq |X|, |X_i| \geq 1\)
7.1.6.3 On multisets

### Syntax 134

```latex
<nValues>
  (<mset> (intVar wspace)2+ </mset>)2+
  [<except> ("{{" intVal ("," intVal)\}+ }" )+ </except>]
  <condition> "(" operator "," operand ")" </condition>
</nValues>
```

### Semantics 95

\[
\text{nValues-mset}(\mathcal{X}, E, (\odot, k)), \text{ with } \mathcal{X} = (X_1, X_2, \ldots), \text{ iff } \\
\left|\left\{\{X_i\} : 1 \leq i \leq |\mathcal{X}| \right\} \cap E \odot k\right|
\]

\[
\text{nValues-mset}(\mathcal{X}, (\odot, k)) \text{ iff } \text{nValues}(\mathcal{X}, \emptyset, (\odot, k))
\]

**Prerequisite**: \(|\mathcal{X}| \geq 2 \land \forall i : 1 \leq i < |\mathcal{X}|, |X_i| = |X_{i+1}| \geq 2 \land \forall M \in E, |M| = |X_1|
```

### 7.2 Constraints lifted to Matrices

Some constraints, introduced earlier on list(s) of integer variables, can naturally be extended to matrices of variables. This means that such constraints restraint both row lists and column lists of variables. The principle is to replace the element `<list>` of the basic constraint by an element `<matrix>`. However, lifting constraints over matrices is not always purely automatic, as we shall show with `cardinality`.

As in the previous section, although we do not introduce new constraint types (names) in XCSP3, we shall refer in the text to the matrix version of a constraint `ctr` by `ctr-matrix`, in order to clarify the textual description (in particular, for the semantics). For example, we shall refer in the text to `allDifferent-matrix` when considering the matrix version of `allDifferent`.

In this section, we present the following constraints defined on matrices of integer variables:

1. `allDifferent-matrix`
2. `ordered-matrix (lex2)`
3. `element-matrix`
4. `cardinality-matrix`

These constraints are defined on matrices of variables. So, they admit a parameter \( \mathcal{M} = [X_1, X_2, \ldots, X_n] \), with \( X_1 = \langle x_{1,1}, x_{1,2}, \ldots, x_{1,m} \rangle \), \( X_2 = \langle x_{2,1}, x_{2,2}, \ldots, x_{2,m} \rangle \), \ldots, given by an element `<matrix>`, assuming here a matrix of size \( n \times m \). Note here that we use square brackets ("[") and "]") to delimit matrices, in order to distinguish them from lists of lists (where angle brackets are used). We use the following notations below:

- \( \forall i : 1 \leq i \leq n, \mathcal{M}[i] = X_i \) denotes the \( i \)th row of \( \mathcal{M} \)
- \( \forall j : 1 \leq i \leq m, \mathcal{M}^T[j] = \langle x_{i,j} : 1 \leq i \leq |\mathcal{M}| \rangle \) denotes the \( j \)th column of \( \mathcal{M} \)
7.2.1 Constraint allDifferent-matrix

The constraint allDifferent-matrix, called alldiffmatrix in [117] and in JaCoP, ensures that the values taken by variables on each row and on each column of a matrix are all different.

**Syntax 135**
```
<allDifferent>
  <matrix> ("(" intVar ("," intVar)+ ")")2+ </matrix>
</allDifferent>
```

**Semantics 96**

allDifferent-matrix(M), with \( M = [X_1, X_2, \ldots, X_n] \), iff

\[
\forall i : 1 \leq i \leq n, \text{allDifferent}(M[i])
\]

\[
\forall j : 1 \leq j \leq m, \text{allDifferent}(M^T[j])
\]

**Prerequisite** : \( \forall i : 1 \leq i \leq n, |X_i| = m \)

**Example 111**
```
<allDifferent>
  <matrix>
    (x1,x2,x3,x4,x5)
    (y1,y2,y3,y4,y5)
    (z1,z2,z3,z4,z5)
  </matrix>
</allDifferent>
```

7.2.2 Constraint ordered-matrix (lex2)

The constraint ordered-matrix, that can be called lex-matrix too, corresponds to lex2 in the literature [52]. It ensures that, for a given matrix of variables, both adjacent rows and adjacent columns are lexicographically ordered. For the syntax, we can use lex instead of ordered, as for ordered-list.

**Syntax 136**
```
<lex>
  <matrix> ("(" intVar ("," intVar)+ ")")2+ </matrix>
  <operator> "lt" | "le" | "ge" | "gt" </operator>
</lex>
```
Semantics 97

\[ \text{lex-matrix}(\mathcal{M}, \odot), \text{ with } \mathcal{M} = [X_1, X_2, \ldots, X_n] \text{ and } \odot = \{<, \leq, \geq, >\}, \text{ iff} \]
\[ \text{lex}(\langle M[1], \ldots, M[n] \rangle, \odot) \]
\[ \text{lex}(\langle M^T[1], \ldots, M^T[m] \rangle, \odot) \]

In the following example, the constraint states that:

\begin{itemize}
  \item \(\langle z_1, z_2, z_3 \rangle \leq_{\text{lex}} \langle z_4, z_5, z_6 \rangle \leq_{\text{lex}} \langle z_7, z_8, z_9 \rangle\)
  \item \(\langle z_1, z_4, z_7 \rangle \leq_{\text{lex}} \langle z_2, z_5, z_8 \rangle \leq_{\text{lex}} \langle z_3, z_6, z_9 \rangle\)
\end{itemize}

Example 112

\[
\text{<lex>}
\text{<matrix>}
(\langle z_1, z_2, z_3 \rangle)
(\langle z_4, z_5, z_6 \rangle)
(\langle z_7, z_8, z_9 \rangle)
\text{</matrix>}
\text{<operator> le </operator>}
\text{</lex>}

7.2.3 Constraint element-matrix

The constraint \text{element-matrix} has been introduced in CHIP and called \text{element-matrix} in the Global Constraint Catalog. Note here that we need to put two variables in \text{<index>} because we need two indexes to designate a variable in the matrix. The optional attributes \text{startRowIndex} and \text{startColIndex} respectively give the numbers used for indexing the first variable in each row and each column of \text{<matrix> (0, by default).}

Syntax 137

\[
\text{<element>}
\text{<matrix [startRowIndex=integer] [startColIndex=integer] >}
("\text{intVar} ("," \text{intVar} + \text{intVar})\)2+
\text{</matrix>}
\text{[<index> intVar \text{wspace} intVar </index>]}
\text{<value> intVal | intVar </value>}
\text{</element>}

Semantics 98

\text{element-matrix}(\mathcal{M}, v), \text{ with } \mathcal{M} = [(x_{1,1}, x_{1,2}, \ldots, x_{1,m}), (x_{2,1}, x_{2,2}, \ldots, x_{2,m}), \ldots], \text{ iff}
\exists (i,j) : 1 \leq i \leq |\mathcal{M}| \land 1 \leq j \leq m \land x_{i,j} = v
\text{element-matrix}(\mathcal{M}, (i, j), v), \text{ with } \mathcal{M} = [(x_{1,1}, x_{1,2}, \ldots, x_{1,m}), (x_{2,1}, x_{2,2}, \ldots, x_{2,m}), \ldots], \text{ iff}
\begin{align*}
x_{i,j} & = v
\end{align*}
7.2.4 Constraint cardinality-matrix

The constraint cardinality-matrix, see [118], ensures a constraint cardinality on each row and each column. For managing cardinality of values, elements <rowOccurs> and <colOccurs> are introduced. Below, we only describe the variant with cardinality variables.

Syntax 138

<table>
<thead>
<tr>
<th>&lt;cardinality&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;matrix&gt; (&quot;intVar &quot;, &quot;intVar+ &quot;)2+ &lt;/matrix&gt;</td>
</tr>
<tr>
<td>&lt;values&gt; (intVal wspace)+ &lt;/values&gt;</td>
</tr>
<tr>
<td>&lt;rowOccurs&gt; (&quot;intVar &quot;, &quot;intVar+ &quot;)2+ &lt;/rowOccurs&gt;</td>
</tr>
<tr>
<td>&lt;colOccurs&gt; (&quot;intVar &quot;, &quot;intVar+ &quot;)2+ &lt;/colOccurs&gt;</td>
</tr>
<tr>
<td>&lt;/cardinality&gt;</td>
</tr>
</tbody>
</table>

Semantics 99

cardinality-matrix(M,V,R,C), with M = [X_1, X_2, ..., X_n], R = (R_1, R_2, ..., R_n)
C = (C_1, C_2, ..., C_m), iff
\forall i: 1 \leq i \leq n, cardinality(M[i], V, R[i])
\forall j: 1 \leq j \leq m, cardinality(M^T[j], V, C[j])

Prerequisite : \forall i: 1 \leq i \leq n, |X_i| = m \land \forall i : 1 \leq i \leq n, |R_i| = |V| \land \forall j : 1 \leq j \leq m, |C_j| = |V|

Example 114

<table>
<thead>
<tr>
<th>&lt;cardinality id=&quot;c&quot;&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;matrix&gt;</td>
</tr>
<tr>
<td>(x1, x2, x3, x4)</td>
</tr>
<tr>
<td>(y1, y2, y3, y4)</td>
</tr>
<tr>
<td>(z1, z2, z3, z4)</td>
</tr>
<tr>
<td>&lt;/matrix&gt;</td>
</tr>
<tr>
<td>&lt;values&gt; 0 1 &lt;/values&gt;</td>
</tr>
<tr>
<td>&lt;rowOccurs&gt; (r10, r11)(r20, r21)(r30, r31) &lt;/rowOccurs&gt;</td>
</tr>
<tr>
<td>&lt;colOccurs&gt; (c10, c11)(c20, c21)(c30, c31)(c40, c41) &lt;/colOccurs&gt;</td>
</tr>
<tr>
<td>&lt;/cardinality&gt;</td>
</tr>
</tbody>
</table>
7.3 Restricted Constraints

A few global constraints are defined as specific restricted cases of other ones; restricting means here hardening. For example, allDifferentSymmetric and increasingNValues are restricted versions of allDifferent and nValues, respectively. Of course, one can always simply post two distinct constraints instead of a combined restricted one in order to get something equivalent. However, it is useful to keep partly the model’s structure and to inform solvers that it is possible to treat combinations of constraints as a single constraint over the same list of variables. That is why we define an attribute restriction, which can be added to any element containing a list of variables, i.e., any element <list>. This attribute can be assigned either a restriction or a list of restrictions (whitespace as separator), chosen among the following list:

- "allDifferent"
- "increasing"
- "strictlyIncreasing"
- "decreasing"
- "strictlyDecreasing"
- "irreflexive"
- "symmetric"
- "convex"

When applied to an element <list>, inside any XCSP3 constraint element ctr, we obtain the following syntax: Note that a possible alternative is to use the meta-constraint and, but the solution we present here is simpler and more compact.

**Syntax 139**

```xml
<ctr>
  ...
  <list restriction="restrictionList"> ... </list>
  ...
</ctr>
```

Because there is usually no ambiguity in the way a restriction applies, a constraint ctr, with restriction res, will be denoted by ctr▷res, as for example allDifferent▷symmetric. The semantics of each restriction type is defined below. For simplicity when introducing the semantics, we assume below that the value of the attribute startIndex, associated with the list (sequence) of variables X, is equal to 1 (although it is 0, by default).

**Semantics 100**

allDifferent(X), with X = ⟨x₁, x₂,...⟩, iff
∀(i,j): 1 ≤ i < j ≤ |X|, xᵢ ≠ xⱼ

increasing(X), with X = ⟨x₁, x₂,...⟩, iff
∀i: 1 ≤ i < |X|, xᵢ ≤ xᵢ₊₁

strictlyIncreasing(X), with X = ⟨x₁, x₂,...⟩, iff
∀i: 1 ≤ i < |X|, xᵢ < xᵢ₊₁
decreasing$(X)$, with $X = \langle x_1, x_2, \ldots \rangle$, iff
\[ \forall i : 1 \leq i \leq |X|, x_i \geq x_{i+1} \]

strictlyDecreasing$(X)$, with $X = \langle x_1, x_2, \ldots \rangle$, iff
\[ \forall i : 1 \leq i \leq |X|, x_i > x_{i+1} \]

irreflexive$(X)$, with $X = \langle x_1, x_2, \ldots \rangle$, iff
\[ \forall i : 1 \leq i \leq |X|, x_i \neq i \]

symmetric$(X)$, with $X = \langle x_1, x_2, \ldots \rangle$, iff
\[ \forall (i,j) : 1 \leq i < j \leq |X|, x_i = j \iff x_j = i \]

convex$(X)$, with $X = \langle x_1, x_2, \ldots \rangle$, iff
\[ \forall v : \min(X) \leq v \leq \max(X), v \in X \]

**Remark 34** If the value of the attribute `startIndex` associated with the list $X$ is denoted by `start`, then the conditions expressed for `irreflexive` and `symmetric` must be of the form $x_i \neq i - 1 + start$.

### 7.3.1 Constraint `allDifferent`+$\bowtie$`symmetric`

The constraint `allDifferent`+$\bowtie$`symmetric` is a classical restriction of `allDifferent`. Not only all variables must take different values, but they must also be either grouped by pairs or left alone. It is thus implicit that the variables cannot be symbolic since values must correspond to indexes. If the $i^{th}$ variable is assigned the value representing element $j$ with $i \neq j$, then the $j^{th}$ variable must be assigned the value representing element $i$. The constraint `allDifferent`+$\bowtie$`symmetric` is expressed by setting the attribute `restriction` of `<list>` to the value "symmetric".

**Semantics 101**

| `allDifferent`+$\bowtie$`symmetric`(X), with $X = \langle x_1, x_2, \ldots \rangle$, iff |
| `allDifferent(X) \land symmetric(X)` |

**Example 115**

```
<allDifferent>
  <list restriction="symmetric"> x1 x2 x3 x4 x5 </list>
</allDifferent>
```

### 7.3.2 Constraint `allDifferent`+$\bowtie$`symmetric+irreflexive`

The constraint `allDifferent`+$\bowtie$`symmetric` can be further restricted by forcing irreflexivity. This is called `oneFactor` in [66]. By forbidding every variable to be assigned the value representing its own index, all variables are necessarily grouped by pairs. The constraint `allDifferent`+$\bowtie$`symmetric+irreflexive` is expressed by setting the attribute `restriction` of `<list>` to the value "symmetric irreflexive".
7.3.3 Constraint allDifferent ⊳ convex

The constraint allDifferent ⊳ convex is called alldifferent_consecutive_values in the Global Constraint Catalog. Not only all variables must take different values, but they must also be consecutive. The constraint allDifferent ⊳ convex is expressed by setting the attribute restriction of <list> to the value "convex".

Semantics 103

\[
\text{allDifferent} \triangleright \text{convex}(X), \text{ with } X = \langle x_1, x_2, \ldots \rangle, \text{ iff } \\
\text{allDifferent} \triangleright \text{convex}(X)
\]

Example 117

\[
\text{<allDifferent>}
\text{<list restriction=\"convex\"> x1 x2 x3 x4 </list>}
\text{</allDifferent>}
\]

7.3.4 Constraint nValues ⊳ increasing

The constraint nValues ⊳ increasing [10] combines nValues and ordered:increasing. It is expressed by setting the attribute restriction of <list> to the value "increasing".

Semantics 104

\[
\text{nValues} \triangleright \text{increasing}(X,(\odot,k)), \text{ with } X = \langle x_1, x_2, \ldots \rangle, \text{ iff } \\
nValues(X,(\odot,k)) \wedge \text{increasing}(X)
\]

In the following example, the constraint c enforces the array of variables X to be in increasing order and to take exactly 2 different values.
7.3.5 Constraint cardinality>increasing

The constraint cardinality>increasing, capturing increasing global cardinality in the Global Constraint Catalog, is the conjunction of cardinality and ordered (with operator ≤). It is obtained by adding "increasing" as restriction to the element <list> of the XCSP3 constraint element <cardinality>.

Semantics 105

\[ \text{cardinality} \triangleright \text{increasing}(X, V, O), \text{with } X = \langle x_1, x_2, \ldots \rangle, \text{iff} \]
\[ \text{cardinality}(X, V, O) \land \text{ordered}(X, \leq) \]

Below, the constraint c enforces the variables \(x_1, x_2, x_3, x_4\) to be in increasing order and to be assigned between 1 and 3 occurrences of value 2 and between 1 and 3 occurrences of value 4. Note that the values taken by these variables do not necessarily have to be in \{2, 4\} since the attribute closed is set to "false".

Example 119

\[ \langle \text{cardinality id="c"} \rangle \]
\[ \langle \text{list restriction="increasing"} \rangle \text{x1 x2 x3 x4} \langle /text/list \rangle \]
\[ \langle \text{values closed="false"} \rangle 2 4 \langle /text/values \rangle \]
\[ \langle \text{occurs} \rangle 1..3 1..3 \langle /text/occurs \rangle \]
\[ \langle /\text{cardinality} \rangle \]

7.3.6 Constraint permutation>increasing (sort)

The constraint permutation>increasing, often called sort in the literature, is the conjunction of permutation and ordered (with operator ≤). It is obtained by adding "increasing" as restriction to the second element <list> of the XCSP3 constraint element <permutation>.

Semantics 106

\[ \text{permutation} \triangleright \text{increasing}(X, Y), \text{with } X = \langle x_1, x_2, \ldots \rangle \text{ and } Y = \langle y_1, y_2, \ldots \rangle, \text{iff} \]
\[ \text{permutation}(X, Y) \land \text{ordered}(Y, \leq) \]
\[ \text{permutation} \triangleright \text{increasing}(X, Y, M) \text{ iff} \]
\[ \text{permutation}(X, Y, M) \land \text{ordered}(Y, \leq) \]

Prerequisite: \(|X| = |Y| = |M| > 1\)
Example 120

<permutation id="c1">
  <list> x1 x2 x3 x4 </list>
  <list restriction="increasing"> y1 y2 y3 y4 </list>
</permutation>

<permutation id="c2">
  <list> v1 v2 v3 </list>
  <list restriction="increasing"> v4 v5 v6 </list>
  <mapping> m1 m2 m3 </mapping>
</permutation>

7.3.7 Constraint sumCosts=allDifferent

The constraint sumCosts=allDifferent [54], also called minimumWeightAllDifferent in the literature, is the conjunction of sumCosts and allDifferent. It is obtained by adding "allDifferent" as restriction to the element <list> of the XCSP3 constraint element <sumCosts>.

Example 121

<sumCosts id="c">
  <list restriction="allDifferent"> y1 y2 y3 </list>
  <costMatrix>
    (10,0,5) // costs for y1
    (0,5,0) // costs for y2
    (5,10,0) // costs for y3
  </costMatrix>
  <condition> (eq,z) </condition>
</sumCosts>
8

Meta-Constraints

In this chapter, we present general mechanisms that can be used to combine constraints. Sometimes in the literature, they are called meta-constraints. We have:

- sliding mechanisms over sequences of variables: slide and seqbin
- logical mechanisms over constraints: and, or, not, ifThen and ifThenElse

They are referred to by <metaConstraint.../> in Chapter 1.

Remark 35 Note that all meta-constraints introduced in this chapter (slide, seqbin, and, or, not, ifThen, ifThenElse) can be reified as they are all considered as single constraints; for reification, see Section 10.3.

8.1 Meta-Constraint slide

A general mechanism, or meta-constraint, that is useful to post constraints on sequences of variables is slide [20]. The scheme slide ensures that a given constraint is enforced all along a sequence of variables. To represent such sliding constraints in XCSP3, we simply build an element <slide> containing a constraint template (for example, one for <extension> or <intension>) to indicate the abstract (parameterized) form of the constraint to be slided, preceded by an element <list> that indicates the sequence of variables on which the constraint must slide. Constraint templates are described in Section 10.1, and possible expressions of <constraint.../> are given in Appendix B. The attribute circular of <slide> is optional ("false", by default); when set to "true", the constraint is slided circularly. The attribute offset of <list> is optional (value 1, by default); it permits, when sliding, to skip more than just one variable of the sequence, capturing slide, in [20].
Syntax 140

```xml
<slide [circular="boolean"]>
  <list [offset="integer"]> (intVar wspace)2+ </list>
  <constraint/></slide>
```

For the semantics, we consider that \( \text{ctr}(\%0,\ldots,\%q-1) \) denotes the template of the constraint \( \text{ctr} \) of arity \( q \), and that \( \text{slide}^{\text{circ}} \) means the circular form of \( \text{slide} \) (i.e., with \( \text{circular}="\text{true}" \) in XCSP3).

Semantics 107

\[
\text{slide}(X,\text{ctr}(\%0,\ldots,\%q-1)), \text{with } X = \langle x_0, x_1, \ldots \rangle, \text{iff}
\]

\[
\forall i: 0 \leq i \leq |X| - q, \text{ctr}(x_i, x_{i+1}, \ldots, x_{i+q-1})
\]

\[
\text{slide}(X, \text{os}, \text{ctr}(\%0,\ldots,\%q-1)), \text{with an offset os}, \text{iff}
\]

\[
\forall i: 0 \leq i \leq (|X| - q)/\text{os}, \text{ctr}(x_i \times \text{os}, x_{i \times \text{os}+1}, \ldots, x_{i \times \text{os}+q-1})
\]

\[
\text{slide}^{\text{circ}}(X, \text{ctr}(\%0,\ldots,\%q-1)) \text{ iff}
\]

\[
\forall i: 0 \leq i \leq |X| - q + 1, \text{ctr}(x_i, x_{i+1}, \ldots, x_{(i+q-1) \mod |X|})
\]

In the following example, \( c_1 \) is the constraint \( x_1 + x_2 = x_3 \land x_2 + x_3 = x_4 \), \( c_2 \) is the circular sliding table constraint \( (y_1, y_2) \in T \land (y_2, y_3) \in T \land (y_3, y_4) \in T \land (y_4, y_1) \in T \) with \( T = \{(a,a), (a,c), (b,b), (c,a), (c,b)\} \) and \( c_3 \) is the sliding \( \neq \) constraint \( w_1 \neq z_1 \land w_2 \neq z_2 \land w_3 \neq z_3 \), with offset 2.

Example 122

```xml
<slide id="c1">
  <list> x1 x2 x3 x4 </list>
  <intension> eq(add(%0,%1),%2) </intension>
</slide>

<slide id="c2" circular="true">
  <list> y1 y2 y3 y4 </list>
  <extension>
    <list>%0 %1</list>
    <supports>(a,a)(a,c)(b,b)(c,a)(c,b)</supports>
  </extension>
</slide>

<slide id="c3">
  <list offset="2"> w1 z1 w2 z2 w3 z3 </list>
  <intension> ne(%0,%1) </intension>
</slide>
```

In some cases, it may be more practical to handle more than one sliding list. This is the reason we can put several successive elements \(<\text{list}\rangle\). In that case, variables are collected from list to list in the order they are put (all variables of a list are considered before starting with the next list). The number of variables to be collected from one list at each iteration is given by the optional attribute \texttt{collect} (value 1, by default). The attribute \texttt{offset} can be associated independently with each list.

The general syntax is:
As an illustration, the following constraint $c_4$ corresponds to $x_1 + y_1 = z_1 \land x_2 + y_2 = z_2 \land x_3 + y_3 = z_3$.

**Example 123**

```
<slide id="c4">
  <list offset="2" collect="2"> x1 y1 x2 y2 x3 y3 </list>
  <list> z1 z2 z3 </list>
  <intension> eq(add(0,1),2) </intension>
</slide>
```

**Remark 36** Note that slide

- can be relaxed/softened, obtaining then cardPath; see Section 9.4.2;
- cannot be a descendant of (i.e., involved in) an element <group>, <slide> or <seqbin>.

### 8.2 Some Classical Uses of slide

#### 8.2.1 Constraint sequence

The constraint `sequence`, see [8, 117], also called `among_seq` in [4], enforces a set of count constraints over a sequence of variables. Although it can be represented by `slide`, we introduce this specific constraint because it is often used. The arity of sliding constraints is given by the attribute `window`.

**Syntax 142**

```
<sequence>
  <list window="integer"> (intVar wspace)2+ </list>
  <values> (intVal wspace)+ </values>
  <condition> "( operator "," operand ")" </condition>
</sequence>
```

For the semantics below, the arity of sliding constraints (i.e., the window size) is given by $q$, the set of values by $V$ and the condition by $(\circ,k)$ (classically, corresponding to an interval $l..u$ of possible cumulated occurrences).

**Semantics 108**

```
sequence(X,q,V,(\circ,k), with X = \langle x_1, x_2, \ldots \rangle, iff
\forall i : 1 \leq i \leq |X| - q + 1, count(\langle x_{i+j} \in X : 0 \leq j < q \rangle, V,(\circ,k))
```

The following constraint
sequence(⟨x₁, x₂, x₃, x₄, x₅⟩, 3, {0, 2, 4}, (∈,0..1))

is then equivalent to:

\[
\text{count}(⟨x₁, x₂, x₃⟩, \{0, 2, 4\}, (∈,0..1)) \\
\wedge \text{count}(⟨x₂, x₃, x₄⟩, \{0, 2, 4\}, (∈,0..1)) \\
\wedge \text{count}(⟨x₃, x₄, x₅⟩, \{0, 2, 4\}, (∈,0..1))
\]

This gives in XCSP3:

**Example 124**

```xml
<sequence id="c">
  <list window="3"> x₁ x₂ x₃ x₄ x₅ </list>
  <values> 0 2 4 </values>
  <condition> (in,0..1) </condition>
</sequence>
```

and with slide:

**Example 125**

```xml
<slide id="c">
  <list> x₁ x₂ x₃ x₄ x₅ </list>
  <count>
    <list> %0 %1 %2 </list>
    <values> 0 2 4 </values>
    <condition> (in,0..1) </condition>
  </count>
</slide>
```

### 8.2.2 Constraint slidingSum

The constraint `slidingSum` [4], enforces a set of sum constraints over a sequence of variables.

For the semantics below, the arity of sliding constraints is given by `q`, and the interval of possible values for the sum by `l..u`.

**Semantics 109**

\[
\text{slidingSum}(X, q, l..u), \text{ with } X = ⟨x₁, x₂, ...⟩, \text{ iff} \\
\forall i : 1 ≤ i ≤ |X|− q + 1, l ≤ \sum_{j=i}^{i+q-1} x_j ≤ u
\]

In the following example, `c` is the sliding sum `1 ≤ x₁ + x₂ + x₃ ≤ 3 \wedge 1 ≤ x₂ + x₃ + x₄ ≤ 3 \wedge 1 ≤ x₃ + x₄ + x₅ ≤ 3`. 

145
8.2.3 Constraints change and smooth

These two constraints are sliding constraints. However, as they also integrate relaxation, we introduce them in Chapter 9.

8.3 Meta-Constraint seqbin

The meta-constraint seqbin \([103, 70]\) ensures that a binary constraint holds down a sequence of variables, and counts how many times another binary constraint is violated.

Syntax 143

```xml
<seqbin>
  <list> (intVar wspace)2+ </list>
  <constraint.../> // template of the hard binary constraint
  <constraint.../> // template of the soft binary constraint
  <number> intVal | intVar </number>
</seqbin>
```

The attribute `violable` is required (with value "true") for the soft binary constraint.

Semantics 110

\[
\text{seqbin}(X, \text{ctr}^h(\%0, \%1), \text{ctr}^s(\%0, \%1), z), \text{ with } X = \langle x_1, x_2, \ldots \rangle, \text{ iff }
\forall i: 1 \leq i < |X|, \text{ctr}^h(x_i, x_{i+1})
\]

Remark 37 Note that in our definition, we do not add 1 to \( z \) as proposed in \([70]\); and implicitly defined in \([103]\). We do believe that it can be easily handled by solvers.

Example 127

```xml
<seqbin id="c1">
  <list> x1 x2 x3 x4 x5 x6 x7 </list>
  <intension> ne (%0 ,%1) </intension>
  <intension violable="true"> lt (%0 ,%1) </intension>
  <number> y </number>
</seqbin>
```
Remark 38  Note that seqbin cannot be a descendant of (i.e., involved in) an element <group>, <slide> or <seqbin>.

8.4 Meta-Constraint and

Sometimes, it may be interesting to combine logically constraints [84, 85]. The meta-constraint and ensures the conjunction of a set of constraints (and possibly, meta-constraints) that are put together inside a same element. This may be useful for obtaining a stronger filtering and/or dealing with reification. Below, <constraint.../> and <metaConstraint.../> represent any constraint and meta-constraint introduced in XCSP3 (see Appendix B for an exhaustive list).

**Syntax 144**

```
<and>
  (<constraint.../>) | (<metaConstraint.../>)+
</and>
```

**Semantics 111**

\[
\text{and(ctr}_1, \text{ctr}_2, ..., \text{ctr}_k), \text{ where } \text{ctr}_1, \text{ctr}_2, ..., \text{ctr}_k \text{ are } k \text{ (meta-)constraints, iff } \\
\text{ctr}_1 \land \text{ctr}_2 \land ... \land \text{ctr}_k
\]

The following example shows a constraint \( c \) that represents \( c_1 \land c_2 \).

**Example 128**

```
<and id="c"
  (<intension id="c1"> eq(x, add(y,z)) </intension>
  (<extension id="c2"> 
    (<list> x z </list>
    (<supports> (0,1)(1,3)(1,2)(1,3)(2,0)(2,2)(3,1) </supports>
  </extension>
</and>
```

Remark 39  Note that and can be relaxed/softened, obtaining then soft-and; see Section 9.4.1.
8.5 Some Classical Uses of and

Some constraints conjunction are classical.

8.5.1 Constraint gen-sequence

A general form of sequence, with name gen-sequence, has been proposed in [137]. It allows specifying the precise sequences of variables on which a constraint count must hold; so, this is no more exactly a sliding constraint. Each such count constraint is defined by a window (interval \( p..q \), giving the indexes of the variables of the consecutive variables of the window) and a range (interval \( l..u \) giving the constraining bounds for the variables of the window).

Semantics 112

\[
\text{gen-sequence}(X, W, V, R), \text{ with } X = \langle x_1, x_2, \ldots \rangle, \text{ } W = \langle p_1..q_1, p_2..q_2, \ldots \rangle, \\
R = \langle l_1..u_1, l_2..u_2, \ldots \rangle, \text{ if } \forall i : 1 \leq i \leq |W|, \text{ count}(\langle x_j \in X : p_i \leq j \leq q_i \rangle, V, (\in, l_i..u_i))
\]

Conditions: \(|W| = |R|

The constraint

\[
\text{gen-sequence}(\langle x_0, x_1, x_2, x_3, x_4 \rangle, \langle 0..2, 2..4 \rangle, \{0, 2, 4\}, \{0, 1, 1..2\})
\]

is equivalent to:

\[
\text{count}(\langle x_0, x_1, x_2 \rangle, \{0, 2, 4\}, (\in, 0..1)) \land \text{count}(\langle x_2, x_3, x_4 \rangle, \{0, 2, 4\}, (\in, 1..2))
\]

In XCSP3, we simply combine the two constraints by and.

Example 129

```
<and>
  <count>
    <list> x0 x1 x2 </list>
    <values> 0 2 4 </values>
    <condition> (in,0..1) </condition>
  </count>
  <count>
    <list> x2 x3 x4 </list>
    <values> 0 2 4 </values>
    <condition> (in,1..2) </condition>
  </count>
</slide>
```

8.5.2 Constraint gsc

The global sequencing constraint (gsc) [120] combines sequence with cardinality. An example is given below.
8.5.3 Constraint cardinalityWithCosts

The constraint cardinalityWithCosts [116] combines cardinality with sumCosts. An example is given below.

Example 131

```xml
<and>
  <cardinality>
    <list id="X"> x1 x2 x3 x4 x5 </list>
    <values> 0 1 2 </values>
    <occurs> y1 y2 y3 </occurs>
  </cardinality>
  <sumCosts>
    <list as="X" />
    <values cost="5"> 0 2 </values>
    <values cost="0"> default </values>
    <condition> (eq ,z) </condition>
  </sumCosts>
</and>
```

8.5.4 Constraints costRegular and multicostRegular

The constraint costRegular [44] combines regular with sumCosts. An example is given below (variables are assumed to have binary domains).

Example 132

```xml
<and>
  <regular>
    <list id="B"> b1 b2 b3 b4 </list>
    <transitions>
      (a,0,a)(a,1,b)(b,0,c)(b,1,d)(c,0,d)
    </transitions>
  </regular>
</and>
```
It is possible, by introducing several constraints `sumCosts` to represent `multicostregular` [92].

### 8.6 Meta-Constraint `or`

The meta-constraint `or` ensures the disjunction of a set of constraints that are put together inside a same element. This may be useful for modeling and/or dealing with reification.

#### Syntax 145

```
<or>
  ((<constraint.../> | <metaConstraint.../>)+
</or>
```

#### Semantics 113

`or(ctr\_1, ctr\_2, ..., ctr\_k)`, where `ctr\_1, ctr\_2, ..., ctr\_k` are `k` (meta-)constraints, iff  

\[ ctr\_1 \lor ctr\_2 \lor ... \lor ctr\_k \]

The following example shows a constraint `c` that represents `c\_1 \lor c\_2`.

#### Example 133

```
<or id="c">
  <intension id="c\_1"> eq(x, add(y, z)) </intension>
  <extension id="c\_2">
    <list> x z </list>
    <supports> (0,1) (1,3) (1,2) (1,3) (2,0) (2,2) (3,1) </supports>
  </extension>
</or>
```

### 8.7 Meta-Constraint `not`

The meta-constraint `not` ensures the negation of a constraint that is put inside the element. This may be useful for modeling and/or dealing with reification.
The meta-constraint `not` can be used to represent well-known global constraints such as `notAllEqual` ensuring that at least one of the involved variables in the constraint `allEqual` must differ from the other ones.

Actually, note that it is possible to represent more directly `notAllEqual` with `nValues`.

### 8.8 Meta-Constraint `ifThen`

The meta-constraint `ifThen` involves two (meta-)constraints. If the first specified one is satisfied, the second one must be satisfied. Below, `<constraint.../>` and `<metaConstraint.../>` represent any constraint and meta-constraint introduced in XCSP3 (see Appendix B for an exhaustive list).

```
Syntax 147
<ifThen>
  (<<constraint.../> | <metaConstraint.../>)/>  // Condition Part
  (<<constraint.../> | <metaConstraint.../>)/>  // Then Part
</ifThen>
```

```
Semantics 115
ifThen(ctr1,ctr2), where ctr1 and ctr2 are two (meta-)constraints, iff
ctr1 ⇒ ctr2
```

The following example shows a meta-constraint `ifThen` that denotes

\[ x = 10 \Rightarrow allDifferent(y) \]

where `y` is an array of variables.
8.9 Meta-Constraint ifThenElse

The meta-constraint ifThenElse involves three (meta-)constraints. If the first specified one is satisfied, the second one must be satisfied. Otherwise, the third one must be satisfied.

Syntax 148

```
<ifThenElse>
  (<constraint.../> | <metaConstraint.../>) // Condition Part
  (<constraint.../> | <metaConstraint.../>) // Then Part
  (<constraint.../> | <metaConstraint.../>) // Else Part
</ifThenElse>
```

Semantics 116

```
ifThenElse(ctr_1, ctr_2, ctr_3), where ctr_1, ctr_2 and ctr_3 are 3 (meta-)constraints, iff
  (ctr_1 ∧ ctr_2) ∨ (¬ctr_1 ∧ ctr_3)
```

The following example shows a meta-constraint ifThenElse that denotes

\[(x = 10 ∧ allDifferent(y)) ∨ (x ≠ 10 ∧ allEqual(y))\]

where \(y\) is an array of variables.
9

Soft Constraints

It is sometimes useful, or even necessary, to express preferences or costs when modeling problems. Actually, many works in the literature focus on the concept of costs that can be associated with certain instantiations of variables inside constraints. There are two main ways of integrating costs:

- by restricting constraints, this way, additionally ensuring that some introduced costs are within certain limits.
- by relaxing constraints, this way, permitting a certain degree of violation with respect to the original forms of constraints;

In Chapters 7 and 8, it was shown how to build restricted constraints, either with the attribute restriction or with the meta-constraint and that enables the integration of the cost-related constraint sumCosts.

In this chapter, we focus on relaxation, and in this context, we introduce cost-based soft constraints. There are two main ways\(^1\) of handling (cost-based) soft constraints. We can deal with hard constraints that are relaxed by integrating cost variables, obtaining so-called relaxed constraints hereafter; see [119, 104, 12]. Or we can deal with cost functions that are also called weighted constraints in the literature (framework WCSP); see [73, 75, 93, 40]. After introducing relaxed constraints and cost functions in Section 9.1, we describe simple relaxation in Section 9.2. In Sections 9.3 and 9.4, complex relaxation for generic constraints and global constraints is presented. Related information concerning WCSP can be found in Section 11.3.1 of Chapter 11.

Although we do not introduce new constraint types (names) in XCSP3, in order to clarify the textual description (and, make semantics unambiguous), we shall refer in the text to a soft variant of a constraint \(\text{ctr}\) by \(\text{soft-ctr}\). For example, we shall refer in the text to \(\text{soft-allDifferent}\) when considering a soft version of \text{allDifferent}.

\(^{1}\)We shall discuss other frameworks for expressing preferences/costs in Chapter 11.
9.1 Relaxed Constraints and Cost Functions

A soft constraint in XCSP3 is an XML constraint element with an attribute type set to the value "soft". A soft constraint in XCSP3 is either a relaxed constraint or a cost function.

**Relaxed Constraints.** In XCSP3, a relaxed constraint is a constraint explicitly integrating a cost component. This means that the XML constraint element contains an element <cost>, which is besides necessarily the last child of the constraint. This new element <cost> can contain a numerical condition (similarly to an element <condition>), which typically involves a cost variable as operand: the actual cost of an instantiation is related to the value of the cost variable with respect to the specified relational operator. The element <cost> can also simply contain an integer variable: in that case, the value of this variable represents exactly the cost of the constraint, and we have intVar which is equivalent to "(eq," intVar ")". As we shall see in the next sections, some optional attributes in mutual exclusion, called violationCost, defaultCost and violationMeasure, can also be specifically introduced.

The syntax is as follows:

```
<constraint type="soft" [violationCost="integer" |
  defaultCost="integer" |
  violationMeasure="measureType"]>
  ...
  <cost> "(" operator "," operand ")" | intVar </cost>
</constraint>
```

**Remark 40** It is important to note that a relaxed constraint remains a hard constraint. Either it is satisfied because the cost condition holds, or it is not.

**Cost Functions.** In XCSP3, a cost function is defined similarly to a relaxed constraint, except that it does not integrate the cost component (element <cost>). A cost function is not a constraint since it returns integer values and not Boolean values.

**Remark 41** It is possible to mix hard constraints with relaxed constraints and cost functions. However, when an XCSP3 instance involves at least one cost function, the attribute type for <instance> must be given the value "WCSP" and no objectives must be present (because the objective function is implicitly defined).

Many illustrations (especially showing the difference between the use of relaxed constraints and cost functions) are given in the next sections.

9.2 Simple Relaxation

The simplest relaxation mechanism consists in associating a fixed integer cost q with some constraints. The cost of the constraint is 0 when the constraint is satisfied, and q otherwise. To deal with simple relaxation, which can be applied to any types of constraints, it suffices to introduce the attribute violationCost whose value must be an integer value.
As an illustration, let us consider a very basic problem: “Lucy, Mary, and Paul want to align in one row for taking a photo. Some of them have preferences next to whom they want to stand: Lucy wants to stand at the left of Mary and Paul also wants to stand at the left of Mary.” Not satisfying Lucy’s preference costs 3 while not satisfying Paul’s preference costs 2. The objective here is to minimize the sum of violation costs.

**Illustration with Relaxed Constraints.** First, let us represent this problem with relaxed constraints. We have three variables for denoting the positions (numbered 1, 2 and 3 from left to right) of Lucy, Mary and Paul in the row. We have a hard constraint \texttt{allDifferent} and two relaxed constraints (note that we need to introduce two variables for representing the violation costs associated with the preferences expressed by Lucy and Paul). Finally, we have an explicit objective: minimizing the sum of the violation costs.

```xml
<instance format="XCSP3" type="COP">
  <variables>
    <var id="lucy"> 1..3 </var>
    <var id="mary"> 1..3 </var>
    <var id="paul"> 1..3 </var>
    <var id="z1"> 0 3 </var>
    <var id="z2"> 0 2 </var>
  </variables>
  <constraints>
    <allDifferent> lucy mary paul </allDifferent>
    <intension type="soft" violationCost="3">
      <function> eq(lucy, sub(mary,1)) </function>
      <cost> z1 </cost>
    </intension>
    <intension type="soft" violationCost="2">
      <function> eq(paul, sub(mary,1)) </function>
      <cost> z2 </cost>
    </intension>
  </constraints>
  <objectives>
    <minimize type="sum"> z1 z2 </minimize>
  </objectives>
</instance>
```

**Illustration with Cost Functions.** Now, let us represent this problem with cost functions. We still have three variables for denoting the positions (numbered 1, 2 and 3 from left to right) of Lucy, Mary and Paul in the row. We have a hard constraint \texttt{allDifferent} and two cost functions for representing the preferences expressed by Lucy and Paul. Note that we do not need to introduce cost variables and to define an explicit objective (WCSP is discussed with more details in Section 11.3.1).
Example 138

<instance format="XCSP3" type="WCSP">
  <variables>
    <var id="lucy"> 1..3 </var>
    <var id="mary"> 1..3 </var>
    <var id="paul"> 1..3 </var>
  </variables>
  <constraints>
    <allDifferent> lucy mary paul </allDifferent>
    <intension type="soft" violationCost="3">
      eq(lucy, sub(mary, 1))
    </intension>
    <intension type="soft" violationCost="2">
      eq(paul, add(lucy, 1))
    </intension>
  </constraints>
</instance>

9.3 Complex Relaxation of Generic Constraints

In this section, we show how to build complex relaxation of the generic constraint types <intension> and <extension>. This relaxation form is more general than the simple one described in Section 9.2 (but dedicated to these two special constraint types). Note that for these two constraint types, we need to modify the way these constraints are classically built: an integer function replaces a Boolean function (predicate), and elements <tuple> replace elements <supports> and <conflicts>, respectively.

9.3.1 Constraint soft-intension

When relaxing an intensional constraint, one has first to replace in <function> the predicate expression (that returns either 0, standing for false, or 1, standing for true) by an integer functional expression (that may return any integer, and should return 0 when the original constraint is satisfied), which is referred to as intExpr in the syntax boxes below, and whose precise syntax is given in Appendix B.

Constraint soft-intension as a Relaxed Constraint. We have to introduce an element <cost>, which gives the following syntax:

Syntax 150

<intension type="soft">
  <function> intExpr </function>
  <cost> "(" operator "," operand ")" | intVar </cost>
</intension>

Below, \( F \) denotes a function (functional expression) with \( r \) formal parameters (not shown here, for simplicity), \( X = \langle x_1, x_2, \ldots, x_r \rangle \) a sequence of \( r \) variables, and \( F(x_1, x_2, \ldots, x_r) \) the
value returned by function $F$ for a given instantiation of variables in $X$. The numerical cost condition is represented by $(\odot, z)$.

**Semantics 117**

| soft-intension($F, X, (\odot, z)$), with $X = \langle x_1, x_2, \ldots, x_r \rangle$, iff $F(x_1, x_2, \ldots, x_r) \odot z$ |

For example, suppose that we want to use $|x - y|$ as cost for the relaxed form of constraint $x = y$, and make this value equal to a cost variable $z$. We obtain constraint $c_1$ below. Now, suppose that we want to use $(v - w)^2$ as cost for the relaxed form of constraint $v \leq w$. Note that we need to be careful about ensuring that the cost is 0 when the constraint is satisfied. Here, the cost must be less than or equal to the value of a cost variable $u$. We obtain constraint $c_2$.

**Example 139**

```xml
<intension id="c1" type="soft">
  <function> dist(x,y) </function>
  <cost> z </cost>
</intension>

<intension id="c2" type="soft">
  <function> if(le(v,w) ,0, sqr(sub(v,w))) </function>
  <cost> (le ,u) </cost>
</intension>
```

**Constraint soft-intension as a Cost Function.** The syntax is given by:

**Syntax 151**

```xml
<intension type="soft">
  <function> intExpr </function>
</intension>
```

The opening and closing tags of `<function>` are optional, which gives:

**Syntax 152**

```xml
<intension type="soft"> intExpr </intension> // Simplified Form
```

For example, suppose that we want to use $|x - y|$ as cost function. We obtain constraint $c_1$ below. Now, suppose that we want to use $(v - w)^2$ as cost function when $v > w$. Again, note that we need to be careful about ensuring that the cost is 0 when the constraint is satisfied. We obtain constraint $c_2$. 

157
9.3.2 Constraint soft-extension

When relaxing an extensional constraint, one has first to refine the enumeration of tuples, by indicating the cost for each of them. The elements <supports> and <conflicts> are then replaced by a sequence of elements <tuples>, where each element <tuples> has a required attribute cost that gives the common cost of all tuples contained inside the element. It is necessary to use the attribute defaultCost of <extension> to specify the cost of all implicit tuples (i.e., those not explicitly listed).

Constraint soft-extension as a Relaxed Constraint. As usual, we have to introduce an element <cost>. This gives the following syntax for extensional constraints of arity greater than or equal to 2:

Syntax 153

```xml
<extension type="soft" defaultCost="integer">
  <list> (intVar wspace)2+ </list>
  (<tuples cost="integer"> (" intVal (" ," intVal)+ ")+ </tuples>)+
  <cost> (" operator "," operand ")" | intVar </cost>
</extension>
```

Semantics 118

`soft-extension(X, T, (0, z))`, with `X = (x₁, x₂, ..., xᵣ)`, `T = (T₁, T₂, ...) where each `Tᵢ` is a set of tuples of cost `cost(Tᵢ)`), iff

```
<x₁, x₂, ..., xᵣ> ∈ Tᵢ ⇒ cost(Tᵢ) ⊗ z
```

Conditions:

- `∀i : 1 ≤ i ≤ |T|, ∀τ ∈ Tᵢ, |τ| = |X|
- `∀(i, j) : 1 ≤ i < j ≤ |T|, Tᵢ ∩ Tⱼ = ∅`
- `∀τ ∈ dom(x₁) × dom(x₂) × ... × dom(xᵣ), ∃i : 1 ≤ i ≤ |T| ∧ τ ∈ Tᵢ`

Assume that we have a relaxed extensional constraint involving three variables `x₁, x₂, x₃`, each of them with domain `{1, 2}`. Assume that the cost for tuples `{1, 2, 2), (2, 1, 2) and (2, 2, 1) is 10, the cost for tuples `{1, 1, 2) and (1, 1, 3) is 5, and the cost for all other tuples—that is, `{1, 1, 1), (1, 2, 1), (2, 1, 1), (2, 2, 2)− is 0. If the cost of an instantiation must be given by a cost variable `z`, we write:
Example 141

```
<extension type="soft" defaultCost="0">
  <list> x1 x2 x3 </list>
  <tuples cost="10"> (1,2,2) (2,1,2) (2,2,1) </tuples>
  <tuples cost="5"> (1,1,2) (1,1,3) </tuples>
  <cost> z </cost>
</extension>
```

For a unary constraint, we obtain:

Syntax 154

```
<extension type="soft" defaultCost="integer">
  <list> intVar </list>
  <tuples cost="integer"> ((intVal | intIntvl) wspace)+ </tuples>
  <cost> "(operator "," operand ")" | intVar </cost>
</extension>
```

Constraint **soft-extension as a Cost Function.** We have the following syntax for cost functions of arity greater than or equal to 2:

Syntax 155

```
<extension type="soft" defaultCost="integer">
  <list> (intVar wspace)2+ </list>
  <tuples cost="integer"> "(" intVal ("," intVal)+ ")"+ </tuples>
</extension>
```

The previous example gives under the form of a cost function:

Example 142

```
<extension type="soft" defaultCost="0">
  <list> x1 x2 x3 </list>
  <tuples cost="10"> (1,2,2) (2,1,2) (2,2,1) </tuples>
  <tuples cost="5"> (1,1,2) (1,1,3) </tuples>
</extension>
```

For a unary cost function, we obtain:

Syntax 156

```
<extension type="soft" defaultCost="integer">
  <list> intVar </list>
  <tuples cost="integer"> ((intVal | intIntvl) wspace)+ </tuples>
</extension>
```
9.4 Complex Relaxation of Global Constraints

In this section, we show how to build complex relaxation of (some) global constraints. This relaxation form is more general than the simple one described in Section 9.2. The way global constraints are defined (i.e., their intern elements or parameters) remains the same, contrary to what we have seen for <intension> and <extension> in Section 9.3.

For global constraints, the attribute violationMeasure, when present, indicates the violation measure used to assess the cost of the constraints. Currently, there are four pre-defined values, but in the future, other values might be introduced too. A default violation measure is considered when this attribute is not present.

It is then possible to soften global constraints by referring to some known violation measures [105, 134]. A violation measure \( \mu \) is simply a cost function that guarantees that cost 0 is associated with, and only with, any tuple that fully satisfies the constraint. Two following violation measures are general-purpose:

- "var": it measures the number of variables that need to change their values in order to satisfy the constraint
- "dec": it measures the number of violated constraints in the binary decomposition of the constraint

Remark 42 In the future, we might introduce an additional attribute violationParameters to provide some information needed by some violation measures. For example, the violation measure "var" can be refined [12] by indicating the subset of variables that are allowed to change their values, when measuring.

Remark 43 The measure "dec", although a general scheme, cannot always be used: for some global constraints, no natural binary decomposition is known.

Let us say a few words about the semantics, when considering relaxed global constraints. For the semantics, we consider that \( \text{soft-ctr} \mu \) is the relaxed form of the global constraint \( \text{ctr} \) of scope \( X \), considering the violation measure \( \mu \). We also consider that \( \mu \) applied to the constraint \( \text{ctr} \) returns the cost for any given instantiation of \( X \), and that \( (\odot, z) \) represents the numerical cost condition.

**Semantics 119**

\[
\text{soft-ctr} \mu \left( X, (\odot, z) \right), \text{ with } X = (x_1, x_2, \ldots, x_r), \text{ iff } \\
\mu(\text{ctr}(x_1, x_2, \ldots, x_r)) \odot z
\]

Finally, when several constraints are at stake, the attribute violationMeasure is usually absent, implicitly referring to the number of constraints that are violated in a set. In the next subsections, we only focus on relaxed global constraints. For global cost functions, it suffices to remove systematically the element <cost>.

9.4.1 Constraint soft-and (Cardinality Operator)

The cardinality operator [133], which must not be confused with the cardinality constraint, connects a cost variable with a set (conjunction) of constraints: the value of the cost variable is the number of violated constraints in the set. In XCSP3, we just need to relax the element <and>; the violation measure is implicit: it counts the number of violated constraints. Note
that we use a violation measure, and not a satisfaction measure; it is immediate to pass from
one form to the other.

Syntax 157
<and type="soft">
  (<constraint.../> | <metaConstraint.../>)+
  <cost> "(" operator "," operand ")" | intVar </cost>
</and>

Semantics 120
soft-and(ctr₁, ctr₂, ..., ctrₖ, (⊙, z)), iff
|{i : 1 ≤ i ≤ k ∧ ¬ctrᵢ}| ⊙ z

An example is given below for a conjunction of three constraints.

Example 143
<and type="soft">
  <intension> eq(w, add(x, y)) </intension>
  <extension>
    <list> v w </list>
    <supports> (0,1)(1,3)(1,2)(1,3)(2,0)(2,2)(3,1) </supports>
  </extension>
  <allDifferent> v w x y </allDifferent>
  <cost> z </cost>
</and>

9.4.2 Constraint soft-slide (cardPath)
A general scheme, or meta-constraint, that is useful to post constraints on sequences of variables
is cardPath [4]. In XCSP3, we just need to relax the element <slide>; the violation measure
is implicit: it counts the number of violated constraints.

Syntax 158
<slide [circular="boolean"] type="soft">
  <list [offset="integer"]> (intVar wspace)+ </list>
  <constraint.../> // constraint template, i.e., constraint involving parameters
  <cost> "(" operator "," operand ")" | intVar </cost>
</slide>

Semantics 121
soft-slide(X, ctr(1, ..., q), (⊙, z)), with X = ⟨x₁, x₂, ...⟩ iff
|{i : 1 ≤ i ≤ |X| − q + 1 ∧ ¬ctr(xᵢ, ..., xᵢ+q−1)}| ⊙ z

161
We first illustrate \texttt{soft-slide} with the constraint \texttt{change} that constrains the number of times an inequality (or another relational operator) holds over a sequence of variables.

<table>
<thead>
<tr>
<th>Semantics 122</th>
</tr>
</thead>
</table>
| \texttt{change}(X, \odot, z), with \( X = (x_1, x_2, \ldots) \) and \( \odot \in \{<,\leq,\geq,=,\neq\} \), iff \[
|\{i: 1 \leq i < |X| \land x_i \odot x_{i+1}\}| = z
| |

Considering that we use a violation measure, and not a satisfaction measure, one can indirectly represent \texttt{change}, as in the following example (where \( z \) is the complementary value of the one given in the semantics).

<table>
<thead>
<tr>
<th>Example 144</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{&lt;slide id=&quot;c1&quot; type=&quot;soft&quot;&gt;}</td>
</tr>
<tr>
<td>\texttt{&lt;list&gt; x1 x2 x3 x4 x5 &lt;/list&gt;}</td>
</tr>
<tr>
<td>\texttt{&lt;intension&gt; ne(0,1) &lt;/intension&gt;}</td>
</tr>
<tr>
<td>\texttt{&lt;cost&gt; z &lt;/cost&gt;}</td>
</tr>
<tr>
<td>\texttt{&lt;/slide&gt;}</td>
</tr>
</tbody>
</table>

We now illustrate \texttt{soft-slide} with the constraint \texttt{smooth} that constrains the number of times a binary constraint comparing a distance between two successive variables and a specified limit holds over a sequence of variables.

<table>
<thead>
<tr>
<th>Semantics 123</th>
</tr>
</thead>
</table>
| \texttt{smooth}(X, l, z), with \( X = (x_1, x_2, \ldots) \), iff \[
|\{i: 1 \leq i < |X| \land |x_i - x_{i+1}| > l\}| = z
| |

Considering that we use a violation measure, and not a satisfaction measure, one can indirectly represent \texttt{smooth}, as in the following example.

<table>
<thead>
<tr>
<th>Example 145</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{&lt;slide id=&quot;c2&quot; type=&quot;soft&quot;&gt;}</td>
</tr>
<tr>
<td>\texttt{&lt;list&gt; y1 y2 y3 y4 &lt;/list&gt;}</td>
</tr>
<tr>
<td>\texttt{&lt;intension&gt; gt(dist(0,1),2) &lt;/intension&gt;}</td>
</tr>
<tr>
<td>\texttt{&lt;cost&gt; w &lt;/cost&gt;}</td>
</tr>
<tr>
<td>\texttt{&lt;/slide&gt;}</td>
</tr>
</tbody>
</table>

Finally, we illustrate \texttt{soft-slide} with the constraint \texttt{softSlidingSum} [4].

<table>
<thead>
<tr>
<th>Semantics 124</th>
</tr>
</thead>
</table>
| \texttt{softSlidingSum}(X, q, l..u, min..max), with \( X = (x_1, x_2, \ldots) \), iff \[
|\{i: 1 \leq i \leq |X| - q + 1 \land l \leq \sum_{j=i}^{i+q-1} x_j \leq u\}| \in \text{min..max}
| |

In the following example, \( c_3 \) is the XCSP3 form of the relaxed sliding sum \( 1 \leq x_1 + x_2 + x_3 \leq 3 \land 1 \leq x_2 + x_3 + x_4 \leq 3 \land 1 \leq x_3 + x_4 + x_5 \leq 3 \) with the constraint that 2 or 3 constraints must be satisfied. Note that the range is 0..1, the complementary of 2..3.
9.4.3 Constraint soft-allDifferent

For the relaxed version [105, 136] of allDifferent, we can use the two general-purpose violation measures "var" and "dec". We only give here the syntax for the relaxed basic variant (and without <except>) of allDifferent.

Syntax 159

```
<allDifferent type="soft" violationMeasure="var|dec">
  <list> (intVar wspace)2+ </list>
  <cost> "(" operator "," operand ")" | intVar </cost>
</allDifferent>
```

For the measure "dec", the semantics is:

Semantics 125

```
soft-allDifferent^{dec}(X,(\odot,z)), with X = \langle x_1, x_2, \ldots \rangle, \odot \in \{<, \leq, >, \geq, =, \neq\}, iff
\left|\{(i,j) : 1 \leq i < j \leq |X| \land x_i = x_j\}\right| \odot z
```

An example is given below:

Example 147

```
<allDifferent type="soft" violationMeasure="dec">
  <list> x1 x2 x3 x4 x5 </list>
  <cost> (le,z) </cost>
</allDifferent>
```

9.4.4 Constraint soft-cardinality

For the soft version of cardinality, we can use "var" as well as the measure "val" defined in [134].
9.4.5 Constraint soft-regular
For the relaxed version of regular, we can use "var" as well as the measure "edit" defined in [134].

9.4.6 Constraint soft-permutation (same)
For the relaxed version of permutation (same), we can use "var" [134], while discarding the optional element <mapping> of the hard version.

9.5 Summary
Soft (cost-based) constraints in XCSP3 can be managed by means of either relaxed constraints or cost functions. On the one hand, it is important to note that relaxed constraints must be understood and treated as hard constraints. Simply, they (typically) involve cost variables which can possibly (i.e., not necessarily) be used for optimization. On the other hand, cost functions necessarily imply an implicit optimization task that consists in minimizing the sum of the constraint costs. Currently, in XCSP3, it is not possible to have both a cost function and an explicit objective function (although extensions in the future can be envisioned). Importantly,
when a cost function is present, the type of the instance is necessarily "WCSP"; see Section 11.3.1.

Finally, a soft constraint is easily identifiable: its attribute type is given the value "soft". It is also easy to determine whether we have a relaxed constraint or a cost function: just check the presence of an element <cost> (as last child). A simple relaxation will necessarily involve the attribute violationCost.
Part IV

Groups, Frameworks and Annotations
Several important features of XCSP3 are introduced in this chapter. In particular, groups of constraints are an essential mechanism to preserve the structure of the problem instances.

### 10.1 Constraint Templates and Groups

A constraint template is a kind of constraint abstraction, that is to say, an element representing a constraint in XCSP3 where some formal parameters are present (typically, for representing missing values and/or variables). Such parameters are denoted by the symbol % followed by a parameter index. A constraint template has \( p \geq 1 \) parameters(s), with indices going from 0 to \( p - 1 \), and of course, a parameter can appear several times.

For example, here is a constraint template, with three parameters, for the element \texttt{intension}:

```
<intension> eq(add(%0,%1),%2) </intension>
```

and another one, with two parameters, for the element \texttt{extension}:

```
<extension>
  <list> %0 %1 </list>
  <supports> (1,2) (2,1) (2,3) (3,1) (3,2) (3,3) </supports>
</extension>
```

A constraint template must be used in a context that permits to furnish the actual parameters, or arguments. A first possibility is within constraints \texttt{slide} and \texttt{seqbin}, where
the arguments are automatically given by the variables of a sequence (sliding effect). A second possibility is to build an element `<group>` whose role is to encapsulate a constraint template followed by a sequence of elements `<args>`. Currently, the constraint template `<constraint.../>` is either put directly inside `<group>`, or put indirectly inside `<group>` through the meta-constraint `<not>`. Each element `<args>` must contain as many arguments as the number of parameters in the constraint template (using whitespace as separator). Of course, the first argument in `<args>` corresponds to `%0`, the second one to `%1`, and so on.

**Remark 44** Do note that a constraint template can only be found in elements `<slide>`, `<seqbin>` and `<group>`, and that no two such elements can be related by an ancestor-descendant relationship.

Considering constraints over integer variables, an argument can be any integer functional expression, referred to as `intExpr` in the syntax box below (its precise syntax is given in Appendix B). When parsing, a solver has to replace the formal parameters of the predicate template by the corresponding arguments to build the constraint.

As a formal parameter, in the context of a group, it is also possible to use `%...` that stands for a variable number of arguments. If `%i` is the formal parameter with the highest index `i` present in the template, then `%...` will be replaced by the sequence (whitespace as separator) of arguments that come after the one associated with `%i`. If `%...` is the only formal parameter present in the template, then `%...` will be replaced by the full sequence of arguments.

**Remark 45** It is currently forbidden to use `%...` inside elements `<slide>` and `<seqbin>`. Also, note that `%...` can never occur in a functional expression (for example, `add(x,%...)` is clearly invalid).

The syntax for the element `<group>`, admitting an optional attribute `id`, is:

```
<group [id="identifier"]>
  <not> <constraint.../> </not> | <constraint.../> // constraint template
  <args> (intExpr wspace)+ </args>+2+
</group>
```

To summarize, an element `<group>` defines a group of constraints sharing the same constraint template. This is equivalent to posting as many constraints as the number of elements `<args>` inside `<group>`. When the attribute `id` is specified for a group, we can consider that the id of each constraint is given by the id of the group followed by `[i]` where `i` denotes the position (starting at 0) of the constraint inside the group. At this point, it should be clear that (syntactic) groups are useful. First, they permit to partially preserve the structure of the problem. Second, there is no need to parse several times the constraint template. Third, the representation is made more compact.

Let us illustrate this important concept of syntactic groups of constraints. The following group of constraints is equivalent to post:

- `g[0]`: `x_0 + x_1 = x_2`
- `g[1]`: `x_3 + x_4 = x_5`
- `g[2]`: `x_6 + x_7 = x_8`

\(^1\)We might extend the possibilities of building groups, in the future.
Example 150

\[
\langle \text{group id="g"} \rangle \\
\langle \text{intension} \rangle \quad \text{eq}(\text{add}(\%0,\%1),\%2) \quad \langle /\text{intension} \rangle \\
\langle \text{args} \rangle \quad x0 \quad x1 \quad x2 \quad \langle /\text{args} \rangle \\
\langle \text{args} \rangle \quad x3 \quad x4 \quad x5 \quad \langle /\text{args} \rangle \\
\langle \text{args} \rangle \quad x6 \quad x7 \quad x8 \quad \langle /\text{args} \rangle \\
\langle /\text{group} \rangle
\]

With \( T = \{(1, 2), (2, 1), (2, 3), (3, 1), (3, 2), (3, 3)\} \), the following group of constraints is equivalent to post:

- \( h[0] \): \((w, x) \in T\)
- \( h[1] \): \((w, z) \in T\)
- \( h[2] \): \((x, y) \in T\)

Example 151

\[
\langle \text{group id="h"} \rangle \\
\langle \text{extension} \rangle \\
\langle \text{list} \rangle \quad \%0 \quad \%1 \quad \langle /\text{list} \rangle \\
\langle \text{supports} \rangle \quad (1, 2)(2,1)(2,3)(3,1)(3,2)(3,3) \quad \langle /\text{supports} \rangle \\
\langle /\text{extension} \rangle \\
\langle \text{args} \rangle \quad w \quad x \quad \langle /\text{args} \rangle \\
\langle \text{args} \rangle \quad w \quad z \quad \langle /\text{args} \rangle \\
\langle \text{args} \rangle \quad x \quad y \quad \langle /\text{args} \rangle \\
\langle /\text{group} \rangle
\]

Now, we give the XCSP3 formulation for the 3-order instance of the Latin Square problem (fill an \( n \times n \) array with \( n \) different symbols, such that each symbol occurs exactly once in each row and exactly once in each column).

Example 152

\[
\langle \text{instance format="XCSP3" type="CSP"} \rangle \\
\langle \text{variables} \rangle \\
\langle \text{array id="x" size="[3][3]"} \rangle \quad 1..3 \quad \langle /\text{array} \rangle \\
\langle /\text{variables} \rangle \\
\langle \text{constraints} \rangle \\
\langle \text{group} \rangle \\
\langle \text{allDifferent} \rangle \quad \%0 \quad \%1 \quad \%2 \quad \langle /\text{allDifferent} \rangle \\
\langle \text{args} \rangle \quad x[0][0] \quad x[0][1] \quad x[0][2] \quad \langle /\text{args} \rangle \\
\langle \text{args} \rangle \quad x[1][0] \quad x[1][1] \quad x[1][2] \quad \langle /\text{args} \rangle \\
\langle \text{args} \rangle \quad x[2][0] \quad x[2][1] \quad x[2][2] \quad \langle /\text{args} \rangle \\
\langle \text{args} \rangle \quad x[0][0] \quad x[1][0] \quad x[2][0] \quad \langle /\text{args} \rangle \\
\langle \text{args} \rangle \quad x[0][1] \quad x[1][1] \quad x[2][1] \quad \langle /\text{args} \rangle \\
\langle \text{args} \rangle \quad x[0][2] \quad x[1][2] \quad x[2][2] \quad \langle /\text{args} \rangle \\
\langle /\text{group} \rangle \\
\langle /\text{constraints} \rangle \\
\langle /\text{instance} \rangle
\]
Note that we can use shorthands for lists of variables taken from arrays, and also the formal parameter %... as illustrated by:

**Example 153**

```xml
<instance format="XCSP3" type="CSP">
  <variables>
    <array id="x" size="[3][3]">1..3</array>
  </variables>
  <constraints>
    <group>
      <allDifferent> %... </allDifferent>
      <args>x[0][]</args>
      <args>x[1][]</args>
      <args>x[2][]</args>
      <args>x[][0]</args>
      <args>x[][1]</args>
      <args>x[][2]</args>
    </group>
  </constraints>
</instance>
```

Of course, in this very special context, a group is not very useful as it is possible to use the constraint `allDifferent-matrix`, leading to:

**Example 154**

```xml
<instance format="XCSP3" type="CSP">
  <variables>
    <array id="x" size="[3][3]">1..3</array>
  </variables>
  <constraints>
    <allDifferent>
      <matrix>x[][]</matrix>
    </allDifferent>
  </constraints>
</instance>
```

As a last illustration, we give the XCSP3 formulation for the 3-order instance of the Magic Square problem. Note that, we cannot replace the element `<group>` by a more compact representation.

**Example 155**

```xml
<instance format="XCSP3" type="CSP">
  <variables>
    <array id="x" size="[3][3]">1..9</array>
  </variables>
  <constraints>
    <allDifferent>x[][]</allDifferent>
    <group>
      <sum>170</sum>
    </group>
  </constraints>
</instance>
```
Remark 46 Note that an element <group>

- cannot be reified, as it is considered as a set of constraints, and not a single one;
- cannot be relaxed/softened, for the same reason;
- can only be a child of <constraints> or a child of an element <block>; hence, it cannot be a descendant of (i.e., involved in) an element <group>, <slide> or <seqbin>.

10.2 Blocks and Classes

We have just seen that we can declare syntactic group of constraints, i.e. groups of constraints built from the same template. But it may be interesting to identify blocks of constraints that are linked together semantically. For example, in a problem model, one may declare a set of constraints that corresponds to clues (typically, for a game), a set of symmetry breaking constraints, a set of constraints related to week-ends when scheduling the shifts of nurses, and so on. This kind of information can be useful for users and solvers. Sometimes, one might also emphasize that there exist some links between variables and constraints.

In XCSP3, there are two complementary ways of managing semantic groups of constraints (and variables): blocks and classes. A block is represented by an XML element whereas a class is represented by an XML attribute. To declare a block of constraints, you just need to introduce an element <block>, with an optional attribute id. Each block may contain several constraints, meta-constraints, groups of constraints and intern blocks. Most of the times, a block will be tagged by one or more classes (a class is simply an identifier), just by introducing the attribute class as in HTML.

Syntax 164

<block [id="identifier"] [class="(identifier wspace)+"]>
  (<constraint.../> | <metaConstraint.../> | <group.../> | <block.../>)+
</block>

Predefined classes are:
- "clues", used for identifying clues or hints usually given for a game,
- "symmetryBreaking", used for identifying elements that are introduced for breaking some symmetries,
- "redundantConstraints", used for identifying redundant (implied) constraints,
- "nogoods", used for identifying elements related to nogood recording

Other predefined values might be proposed later. It is also possible to introduce user-defined classes (i.e., any arbitrary identifier), making this approach very flexible.

An illustration, we give the skeleton of an element \(<\text{constraints}>\) that contains several blocks. A first block contains the constraints corresponding to some clues (for example, the initial values of a Sudoku grid). A second block introduces some symmetry breaking constraints (\(<\text{lex}>\)). A third block introduces some redundant constraints. And finally, the two last blocks refer to constraints related to the management of two different weeks; by introducing blocks here, we can associate a note (short comment) with them.

```
Example 156
<constraints>
  <block class="clues">
    <intension> ... </intension>
    <intension> ... </intension>
    ...
  </block>
  <block class="symmetryBreaking">
    <lex> ... </lex>
    <lex> ... </lex>
    ...
  </block>
  <block class="redundantConstraints"/>
  <block note="Management of first week">
  </block>
  <block note="Management of second week">
  </block>
</constraints>
```

Remark 47 The class "redundantConstraints" permits to identify implied constraints that are usually posted in order to improve the solving process. Because they are properly identified (by means of the attribute class), a solver can be easily asked to discard them, so as to compare its behavior when solving an instance with and without the redundant constraints. Note that this facility can be used with any value of class.

Remark 48 The attribute type of <block>, introduced in previous specifications, is deprecated.

10.3 Reification

Reification of a (hard) constraint \(c\) means associating a 0/1 variable \(x_c\) with \(c\) such that \(x_c\) denotes the truth value of \(c\). Reification can be stated as:

\[ x_c \Leftrightarrow c \]

Reification of \(c\) through \(x_c\) means that:
• \( c \) must be true if \( x_c \) is set to 1
• \( \neg c \) must be true if \( x_c \) is set to 0
• \( x_c \) must be set to 1 if \( c \) becomes entailed
• \( x_c \) must be set to 0 if \( c \) becomes disentailed

Half reification uses implication instead of equivalence. Half reification can be stated as:

\[
x_c \Rightarrow c
\]

Half reification of \( c \) orientated from \( x_c \) means that:
• \( c \) must be true if \( x_c \) is set to 1
• \( x_c \) must be set to 0 if \( c \) becomes disentailed
Half reification can also be stated as:

\[
x_c \Leftarrow c
\]

Half reification of \( c \) orientated towards \( x_c \) means that:
• \( \neg c \) must be true if \( x_c \) is set to 0
• \( x_c \) must be set to 1 if \( c \) becomes entailed

In XCSP3, to reify a constraint, you just need to specify an attribute \texttt{reifiedBy} whose value is the id of a 0/1 variable, to be associated with the XCSP3 element defining this constraint. For half reification, we use either the attribute \texttt{hreifiedFrom} (for \( x_c \Rightarrow c \)) or the attribute \texttt{hreifiedTo} (for \( x_c \Leftarrow c \)).

As an example, taken from [51], let us suppose that you need to express the following constraint:

\[
x \leq 4 \Rightarrow t[x] \times y \geq 6
\]

which requires that if \( x \leq 4 \) then the value at the \( x \)th position of array \( t \) (assumed here to be \([2, 5, 3, 1, 4]\)) multiplied by \( y \) must be at least 6. By introducing variables \( b_1, b_2 \) for reification and variable \( z \) for handling array indexing, we obtain:

\[
c_1: b_1 \Leftrightarrow x \leq 4
\]
\[
c_2: \text{Element}(t, x, z)
\]
\[
c_3: b_2 \Leftrightarrow z \times x \geq 6
\]
\[
c_4: b_1 \Rightarrow b_2
\]

This gives in XCSP3 form:

```xml
Example 157
<instance format="XCSP3" type="CSP">
    <variables>
        <var id="x"> 0..10 </var>
        <var id="y"> 0..10 </var>
        <var id="z"> 1..5 </var>
        <var id="b1"> 0 1 </var>
        <var id="b2"> 0 1 </var>
    </variables>
    <constraints>

<xcsp3>
```

Another example taken from [51] is:
\[ x > 4 \lor \text{allDifferent}(x, y, y - x) \]

With reification, we should have a propagator for the negation of \text{allDifferent}. With half reification, we obtain:

\begin{align*}
  c_1 &: b_1 \Rightarrow x > 4 \\
  c_2 &: z = y - x \\
  c_3 &: b_2 \Rightarrow \text{allDifferent}(x, y, z) \\
  c_4 &: b_1 \lor b_2
\end{align*}

This gives in XCSP3 form:

\begin{verbatim}
Example 158
<instance format="XCSP3" type="CSP">
  <variables>
    <var id="x"> 0..10 </var>
    <var id="y"> 0..10 </var>
    <var id="z"> -10..10 </var>
    <var id="b1"> 0 1 </var>
    <var id="b2"> 0 1 </var>
  </variables>
  <constraints>
    <intension id="c1" hreifiedFrom="b1"> gt(x,4) </intension>
    <intension id="c2"> eq(z,sub(y,x)) </intension>
    <allDifferent id="c3" hreifiedFrom="b2"> x y z </allDifferent>
    <intension id="c4"> or(b1,b2) </intension>
  </constraints>
</instance>
\end{verbatim}

Note that it is also possible to post a constraint \text{allDifferent} containing \text{x y sub(y,x)}, as shown in the next section.

10.4 Views

XCSP3 allows the user to express general formulations of constraints, typically by permitting the use of functional expressions where a variable is usually expected. Some constraint solvers
are able to cope directly with such formulations through the concept of views [28]. A variable view is a kind of adaptor that performs some transformations when accessing the variable it abstracts over. For example, suppose that you have a filtering algorithm (propagator) for the constraint \texttt{allDifferent}. Can you directly deal with \texttt{allDifferent}((x_1+1, x_2+2, x_3+3)), or must you introduce intermediate variables? In XCSP3, we let the possibility of generating instances that are “view-compatible”. As a classical illustration, let us consider the 8-queens problem instance. We just need 8 variables \(x_0, x_1, \ldots, x_7\) and ensure that the values of all \(x_i\) variables, the values of all \(x_i - i\), and the values of all \(x_i + i\) must be pairwise different.

This leads to a XCSP3 instance composed of an array of 8 variables and three \texttt{allDifferent} constraints.

```
例 159
<instance format="XCSP3" type="CSP">
<variables>
  <array id="x" size="[8]">1..8</array>
</variables>
<constraints>
  <allDifferent id="rows">
  </allDifferent>
  <allDifferent id="diag1">
    add(x[0],0) add(x[1],1) add(x[2],2) add(x[3],3)
    add(x[4],4) add(x[5],5) add(x[6],6) add(x[7],7)
  </allDifferent>
  <allDifferent id="diag2">
    sub(x[0],0) sub(x[1],1) sub(x[2],2) sub(x[3],3)
    sub(x[4],4) sub(x[5],5) sub(x[6],6) sub(x[7],7)
  </allDifferent>
</constraints>
</instance>
```

10.5 Aliases (attribute \texttt{as})

In some cases, it is not possible to avoid some redundancy (similar contents of elements), even when using the mechanisms described above. This is the reason why we have a mechanism of aliases. It is implemented by an attribute \texttt{as} that can be used by any element, anywhere in the document, to refer to the content of another element of the document. The semantics is the following: the content of an element <\texttt{elt}> with attribute \texttt{as} set to value "idOther" is defined as being the content of the element <\texttt{eltOther}> in the document with attribute \texttt{id} set to value "idOther". There are a few restrictions:

- the element <\texttt{elt}> must not contain anything of its own,
- the element <\texttt{eltOther}> must precede <\texttt{elt}> in the document,
- the element <\texttt{eltOther}> cannot contain an element equipped with the attribute \texttt{id}.
- the element <\texttt{eltOther}> cannot be specified an attribute \texttt{as} (no allowed transitivity)

Let us illustrate this. Suppose that an instance must contain variables \(x_0, x_1, x_2, x_3\), with domain \(\{a, b, c\}\) for \(x_0\) and \(x_2\), and domain \(\{a, b, c, d, e\}\) for \(x_1\) and \(x_3\). If for some reasons, you
want to preserve the names of the variables, you have to write:

```xml
Example 160
<variables>
    <var id="x0"> a b c </var>
    <var id="x1"> a b c d e </var>
    <var id="x2"> a b c </var>
    <var id="x3"> a b c d e </var>
</variables>
```

By using the attribute `as`, you obtain the following equivalent non-redundant form:

```xml
Example 161
<variables>
    <var id="x0"> a b c </var>
    <var id="x1"> a b c d e </var>
    <var id="x2" as="x0" />
    <var id="x3" as="x1" />
</variables>
```

Of course, this can be applied to any kind of elements. For example, if for some reason, you describe twice the same set of tuples in the document, like for example:

```xml
Example 162
<constraints>
    ...
    <supports> (a,b)(a,c)(b,a)(b,c)(c,a)(c,c) </supports>
    ...
    <conflicts> (a,b)(a,c)(b,a)(b,c)(c,a)(c,c) </conflicts>
    ...
</constraints>
```

then, you can simply write (note that we have introduced here the attribute `id` that is optional for certain elements):

```xml
Example 163
<constraints>
    ...
    <supports id="tps"> (a,b)(a,c)(b,a)(b,c)(c,a)(c,c) </supports>
    ...
    <conflicts as="tps" />
    ...
</constraints>
```
In this chapter, we show how to define instances in various CP frameworks.

11.1 Dealing with Satisfaction (CSP)

A discrete Constraint Network (CN) \( P \) is a pair \((\mathcal{X}, \mathcal{C})\) where \( \mathcal{X} \) denotes a finite set of variables and \( \mathcal{C} \) denotes a finite set of constraints. A CN is also called a CSP instance.

To define a CSP instance, in XCSP3, you have to:

- set the attribute type of <instance> to "CSP";
- enumerate variables (at least one) within <variables>;
- enumerate constraints within <constraints>;

The syntax is as follows:

```
<instance format="XCSP3" type="CSP">
  <variables>
    (<var.../> | <array.../>)+
  </variables>
  <constraints>
    (<constraint.../> | <metaConstraint.../> | <group.../> | <block.../>)*
  </constraints>
  [<annotations.../>]
</instance>
```

As an illustration, here is a way to represent the instance langford-2-04; see CSPLib.
11.2 Dealing with Optimization (COP)

A COP instance is defined by a set of variables $\mathcal{X}$, a set of constraints $\mathcal{C}$, as for a CN, together with a set of objective functions $\mathcal{O}$. Mono-objective optimization is when only one objective function is present in $\mathcal{O}$. Otherwise, this is multi-objective optimization.

To define a COP instance, in XCSP3, you have to:

- set the attribute type of <instance> to "COP";
- enumerate variables (at least one) within <variables>;
- enumerate constraints (if any) within <constraints>;
- enumerate objectives (at least one) within <objectives>.

The syntax is as follows:

```xml
<instance format="XCSP3" type="COP">
  <variables>
    (<var.../> | <array.../>)+
  </variables>
  <constraints>
    (<constraint.../> | <metaConstraint.../> | <group.../> | <block.../>)*
  </constraints>
  <objectives [combination="combinationType"]>
    (<minimize.../> | <maximize.../>)+
  </objectives>
  [<annotations.../>]
</instance>
```

As an illustration, let us consider the Coins problem: what is the minimum number of coins that allows one to pay exactly any price $p$ smaller than one euro [3]. Here, we consider the instance of this problem for $p = 83.$
Remark 49 MaxCSP is an optimization problem, which consists in satisfying the maximum number of constraints of a given CN. Typically, you may be interested in that problem, when CSP instances are over-constrained, i.e., without any solution. In the format, we do not need to refer to MaxCSP: simply feed your solver with (unsatisfiable) CSP instances, and activate MaxCSP solving.

11.3 Dealing with Preferences through Soft Constraints

The classical CSP framework can be extended by introducingvaluations to be associated with constraint tuples [24], making it possible to express preferences. We show below how two main specializations of the Valued Constraint Satisfaction Problem (VCSP) are represented in XCSP3.

11.3.1 WCSP

WCSP (Weighted CSP) is an extension to CSP that relies on a specific valuation structure $S(k) = ([0, \ldots, k], \oplus, \geq)$ where:

- $k \in [1, \ldots, \infty]$ is either a strictly positive natural or infinity,
- $[0, 1, \ldots, k]$ is the set of naturals less than or equal to $k$,
- $\oplus$ is the sum over the valuation structure defined as: $a \oplus b = \min\{k, a + b\}$,
- $\geq$ is the standard order among naturals.

A Weighted Constraint Network (WCN), or WCSP instance, is defined by a valuation structure $S(k)$, a set of variables, and a set of weighted constraints, also called cost functions (see Chapter 9). A WCN is also known as a CFN (Cost Function Network).
Some examples of cost functions, taken from Chapter 9 are given below:

Example 166

```xml
<extension type="soft" defaultCost="0">
  <list> x1 x2 x3 </list>
  <tuples cost="10"> (1,2,2) (2,1,2) (2,2,1) </tuples>
  <tuples cost="5"> (1,1,2) (1,1,3) </tuples>
</extension>
<intension type="soft" violationCost="3">
  eq(lucy, sub(mary,1))
</intension>
<intension type="soft"> dist(x,y) </intension>
<allDifferent type="soft" violationMeasure="dec">
  x1 x2 x3 x4 x5
</allDifferent>
```

Remark 50 When representing a WCSP instance, it is possible to refer to hard constraints. For such constraints, an allowed tuple has a cost of 0 while a disallowed tuple has a cost equal to the value of $k$.

To define a CFN (WCSP instance), in XCSP3, you have to:

- set the attribute `type` of `<instance>` to "WCSP";
- enumerate variables (at least one) inside `<variables>`;
- optionally set a value to the optional attribute `ub` of the element `<constraints>`; this value represents $k$ in the WCSP valuation structure and must be an integer greater than or equal to 1, or the special value "+infinity"; the default value for `ub` is "+infinity";
- optionally set a value to the optional attribute `lb` of the element `<constraints>`. If present, this value must be an integer greater than or equal to 0, and less than or equal to the value of `ub`. It represents a constant cost that must be added to the other costs, sometimes called the 0-ary constraint of the WCSP framework (e.g. see [74]). Of course, if it is not present, it is assumed to be equal to 0;
- enumerate hard constraints, relaxed constraints and cost functions (at least, one) inside `<constraints>`.

The syntax is as follows:

Syntax 167

```xml
<instance format="XCSP3" type="WCSP">
  <variables>
    ( <var.../> | <array.../> )+
  </variables>
  <constraints [lb="integer"] [ub="integer|+infinity"]>
    ( <constraint .../> | <constraint type="soft".../> | <group.../> | <block.../> )*
  </constraints>
</instance>
```
In the simple example below, we have a CFN with two variables, two unary extensional constraints and one binary extensional constraint. Note that the forbidden cost limit is 5 (value of ub).

Example 167

```xml
<instance format="XCSP3" type="WCSP">
  <variables>
    <var id="x"> 0..3 </var>
    <var id="y"> 0..3 </var>
  </variables>
  <constraints ub="5">
    <extension type="soft" defaultCost="0">
      <list>x</list>
      <tuples cost="1"> 1 3 </tuples>
    </extension>
    <extension type="soft" defaultCost="0">
      <list>y</list>
      <tuples cost="1"> 1 2 </tuples>
    </extension>
    <extension type="soft" defaultCost="0">
      <list>x y</list>
      <tuples cost="5">(0,0),(0,1),(1,0),(1,1),(1,2),(2,1),(2,2),(2,3),(3,2),(3,3)</tuples>
    </extension>
  </constraints>
</instance>
```

### 11.3.2 FCSP

A fuzzy constraint network (FCN) is composed of a set of variables and a set of fuzzy constraints. Each fuzzy constraint represents a fuzzy relation on its scope: it associates a value in [0,1] (either a rational, a decimal or one of the integers 0 and 1), called membership degree, with each constraint tuple \( \tau \), indicating to what extent \( \tau \) belongs to the relation and therefore satisfies the constraint. The membership degree of a tuple gives us the preference for that tuple. In fuzzy constraints, preference 1 is the best one and preference 0 the worst one.

In XCSP3, a fuzzy constraint is systematically represented by an XML element whose attribute `type` is set to "fuzzy". Below, we show how to represent fuzzy constraints, either in extensional form, or in intensional form.

**Fuzzy Constraints in Extension.** For representing such a constraint, an element `<extension>` with the attribute `type` set to "fuzzy" is used, containing an element `<list>` and a sequence of elements `<tuples>`. Each element `<tuples>` has a required attribute `degree` that gives the common membership degree of all tuples contained inside the element. It is possible to use
the optional attribute `defaultDegree` of `<extension>` to specify the membership degree of all implicit tuples (i.e., those not explicitly listed).

This gives the following syntax for fuzzy extensional constraints of arity greater than or equal to 2:

**Syntax 168**

```
<extension type="fuzzy" [defaultDegree="number"]>
  <list> (intVar wspace)2+ </list>
  (<tuples degree="number"> (" intVal ("," intVal)+ ")+ </tuples>)+
</extension>
```

An example is given below.

**Example 168**

```
<extension id="c1" type="fuzzy" defaultDegree="0">
  <list> x y z </list>
  <tuples degree="0.1"> (1,2,3,4)(2,1,2,3)(3,1,3,4) </tuples>
  <tuples degree="0.6"> (1,1,3,4)(2,2,2,2) </tuples>
</extension>
```

For a fuzzy unary constraint, we obtain:

**Syntax 169**

```
<extension type="fuzzy" [defaultDegree="number"]>
  <list> intVar </list>
  (<tuples degree="number"> ((intVal | intIntvl) wspace)+ </tuples>)+
</extension>
```

**Fuzzy Constraints in Intension.** For representing such a constraint, we need an element `<intension>` with the attribute `type` set to "fuzzy". It contains a functional expression representing a fuzzy relation returning a real value (comprised between 0 and 1).

**Syntax 170**

```
<intension type="fuzzy">
  <function> realExpr </function>
</intension>
```

The opening and closing tags of `<function>` are optional, which gives:

**Syntax 171**

```
<intension type="fuzzy"> realExpr </intension>  // Simplified Form
```

An example is given below.
To define a FCSP instance, in XCSP3, you have to;
• set the attribute type of <instance> to "FCSP",
• enumerate variables (at least one) within <variables>;
• enumerate fuzzy constraints within <constraints>;

The syntax is as follows:

As an illustration, let us consider the XCSP3 representation of the problem instance described in [50], page 110.

Example 170

```xml
<instance format="XCSP3" type="FCSP">
  <variables>
    <var id="x"> 0..7 </var>
    <var id="y"> 0..7 </var>
    <var id="z"> 0..7 </var>
  </variables>
  <constraints>
    <extension id="c0" type="fuzzy" defaultDegree="0.25">
      <list> x </list>
      <tuples degree="1"> 4 </tuples>
      <tuples degree="0.75"> 3 5 </tuples>
    </extension>
    <extension id="c1" type="fuzzy" defaultDegree="0.5">
      <list> y </list>
      <tuples degree="1"> 3 4 </tuples>
    </extension>
    <extension id="c2" type="fuzzy" defaultDegree="0">
      <list> z </list>
      <tuples degree="1"> 2 </tuples>
      <tuples degree="0.75"> 1 3 </tuples>
    </extension>
  </constraints>
</instance>
```
11.4 Dealing with Quantified Variables

Two frameworks for dealing with quantification of variables have been introduced in the literature. The former, QCSP+, is an extension of CSP, and the latter, QCOP+, is an extension of COP.

11.4.1 QCSP(+)

The Quantified Constraint Satisfaction Problem (QCSP) is an extension of CSP in which variables may be quantified universally or existentially. A QCSP instance corresponds to a sequence of quantified variables, called prefix, followed by a conjunction of constraints. QCSP and its semantics were introduced in [25]. QCSP+ is an extension of QCSP, introduced in [13] to overcome some difficulties that may occur when modeling real problems with classical QCSP. From a logic viewpoint, an instance of QCSP+ is a formula in which (i) quantification scopes of alternate type are nested one inside the other, (ii) the quantification in each scope is restricted by a CSP called restriction or precondition, and (iii) a CSP to be satisfied, called goal, is attached to the innermost scope. An example with 4 scopes is:

\[
\forall X_1 \ (L_1^0(X_1) \rightarrow \\
\exists Y_1 \ (L_1^1(X_1,Y_1) \land \\
\forall X_2 \ (L_2^1(X_1,Y_1,X_2) \rightarrow \\
\exists Y_2 \ (L_2^2(X_1,Y_1,X_2,Y_2) \land \ G(X_1,X_2,Y_1,Y_2))
\)]

(11.1)

where \(X_1, X_2, Y_1,\) and \(Y_2\) are in general sets of variables, and each \(L_i^Q\) is a conjunction of constraints. A more compact and readable syntax for QCSP+ employs square braces to enclose restrictions. An example with 3 scopes is as follows:

\[
\forall X_1[L_1^0(X_1)] \exists Y_1[L_1^1(X_1,Y_1)] \forall X_2[L_2^2(X_1,Y_1,X_2)] \ G(X_1,Y_1,X_2)
\]

which reads "for all values of \(X_1\) which satisfy the constraints \(L_1^0(X_1)\), there exists a value for \(Y_1\) that satisfies \(L_1^1(X_1,Y_1)\) and is such that for all values for \(X_2\) which satisfy \(L_2^2(X_1,Y_1,X_2)\), the goal \(G(X_1,Y_1,X_2)\) is satisfied".

A standard QCSP can be viewed as a particular case of QCSP+ in which all quantifications are unrestricted, i.e. all the CSPs \(L_i^Q\) are empty.

---

\(^1\)This is a revised version of the XCSP 2.1 proposal made initially for QCSP and QCSP+ by M. Benedetti, A. Lallouet and J. Vautard.
To define a QCSP/QCSP\(^+\) instance, in XCSP3, you have to:

- set the attribute type of \(<\text{instance}?>\) to "QCSP" or "QCSP+";
- enumerate variables (at least one) within \(<\text{variables}?>\);
- enumerate constraints within \(<\text{constraints}?>\);
- handle quantification in an element \(<\text{quantification}?>\).

**Handling Quantification.** The element \(<\text{quantification}?>\) provides an ordered list of quantification blocks. Notice that the order in which quantification blocks are listed inside this XML element provides key information, as it specifies the left-to-right order of (restricted) quantifications associated with the QCSP/QCSP\(^+\) instance. Each block of a QCSP/QCSP\(^+\) instance is represented by either an element \(<\text{forall}?>\) or an element \(<\text{exists}?>\), which gives the kind of quantification for all the variables in the block. For QCSP, the list of variables of a block is given directly inside elements \(<\text{forall}?>\) and \(<\text{exists}?>\). However, for QCSP\(^+\), if restrictions are present, we put the ids of restrictions (restricting constraints) inside an element \(<\text{ctrs}?>\), and put the list of variables inside an element \(<\text{vars}?>\); note that restrictions are put with other constraints in \(<\text{constraints}?>\). At least one variable must be present in each block, and the order in which variables are listed is not relevant.

**Remark 51** Each variable can be mentioned in at most one block. Each variable must be mentioned in at least one block; this means the problem is closed, i.e. no free variables are allowed\(^2\).

For QCSP, the syntax is as follows:

\[
\text{Syntax 173}
\]

\[
<\text{instance format=\"XCSP3\" type=\"QCSP\">}
<\text{variables}>
\quad (\text{<var.../> } | \text{<array.../>})+
</\text{variables}>
<\text{constraints}>
\quad (\text{<constraint.../> } | \text{<metaConstraint.../> } | \text{<group.../> } | \text{<block.../>})*
</\text{constraints}>
<\text{quantification}>
\quad (\text{<exists> } (\text{intVar wspace})+ \text{<exists> } | \text{<forall> } (\text{intVar wspace})+ \text{<forall>})+
</\text{quantification}>
[\text{<annotations.../>}]
</\text{instance}>
\]

Let \(w, x, y\) and \(z\) be four variables, whose domains are \(\{1, 2, 3, 4\}\). An XCSP3 encoding of the QCSP:

\[
\exists w, x \ \forall y \ \exists z \quad w + x = y + z, \quad y \neq z
\]

is given by:

\(^2\)This restriction may be relaxed by future formalizations of open QCSPs.
Recall that, in XCSP3, an \textit{intVar} corresponds to the id of a variable declared in \textit{<variables>}. Similarly, an \textit{intCtr} corresponds to the id of a constraint declared in \textit{<constraints>}. For QCSP\(^{+}\), the syntax is then as follows:

An XCSP3 encoding of the QCSP\(^{+}\): 

\[ \exists w, x \left[ w + x < 8, w - x > 2 \right] \forall y \left[ w \neq y, x \neq y \right] \exists z \left[ z < w - y \right] w + x = y + z \]

is given by:

```xml
<instance format="XCSP3" type="QCSP">
  <variables>
    <var id="w"> 1..4 </var>
    <var id="x"> 1..4 </var>
    <var id="y"> 1..4 </var>
    <var id="z"> 1..4 </var>
  </variables>
  <constraints>
    <intension> eq(add(w,x),add(y,z)) </intension>
    <intension> ne(y,z) </intension>
  </constraints>
  <quantification>
    <exists> w x </exists>
    <forall> y </forall>
    <exists> z </exists>
  </quantification>
</instance>
```
Example 172

```xml
<instance format="XCSP3" type="QCSP+">
  <variables>
    <var id="w"> 1..4 </var>
    <var id="x"> 1..4 </var>
    <var id="y"> 1..4 </var>
    <var id="z"> 1..4 </var>
  </variables>
  <constraints>
    <intension id="r1a"> lt(add(w,x),8) </intension>
    <intension id="r1b"> gt(sub(w,x),2) </intension>
    <intension id="r2a"> ne(w,y) </intension>
    <intension id="r2b"> ne(x,y) </intension>
    <intension id="r3"> gt(sub(w,y),z) </intension>
    <intension id="goal"> eq(add(w,x),add(y,z)) </intension>
  </constraints>
  <quantification>
    <exists>
      <vars> w x </vars>
      <ctrs> r1a r1b </ctrs>
    </exists>
    <forall>
      <vars> y </vars>
      <ctrs> r2a r2b </ctrs>
    </forall>
    <exists>
      <vars> z </vars>
      <ctrs> r3 </ctrs>
    </exists>
  </quantification>
</instance>
```

Note here that we have to specify an id for each restricting constraint in order to be able to reference them.

11.4.2 QCOP(+)

QCOP(+), Quantified Constraint Optimization problem, is a framework [14] that allows us to formally express preferences over QCSP(+) strategies. A QCOP(+) instance is obtained from a QCSP(+) instance by adding preferences and aggregates to the quantification blocks. For aggregation, we need aggregate ids (which look like local variables but cannot be part of constraints) and aggregate functions. Possible aggregate functions are:

- "sum",
- "product"
- "average"
- "deviation"
- "median"
- "count"
Compared to QCSP(+), we must add information to elements `<exists>` and `<forall>` for dealing with optimization. For an element `<exists>`, we need an optimization condition, which is represented by an element `<minimize>` or an element `<maximize>`, the content of which is a variable id or an aggregate id. However, when no element `<minimize>` or `<maximize>` is present for an element `<exists>`, this implicitly corresponds to the atom `any` [14]. For an element `<forall>`, we must add a sequence of one or several elements `<aggregate>`, each of them with a required attribute id that gives the aggregate identifier. The content of the element is an expression of the form `f(x)` where `f` denotes an aggregate function and `x` denotes a variable id or an aggregate id.

To define a QCOP/QCOP+ instance, in XCSP3, you have to:
- set the attribute `type` of `<instance>` to "QCOP" or "QCOP+";
- enumerate variables (at least one) within `<variables>`;
- enumerate constraints (at least one) within `<constraints>`;
- handle quantification in an element `<quantification>`.

As an illustration, here is the XCSP3 representation of the toy problem introduced in [14], page 472. We assume here that the array `A`, given as an input to the instance is `{12, 5, 6, 9, 4, 3, 13, 10, 12, 5}`.

Example 173

```
<instance format="XCSP3" type="QCOP+">
    <variables>
        <var id="x"> 0 1 </var>
        <var id="i"> 0..9 </var>
        <var id="z"> 0..+infinity </var>
    </variables>
    <constraints>
        <intension id="res"> eq(mod(i,2),x) </intension>
        <element id="goal">
            <list> 12 5 6 9 4 3 13 10 12 5 </list>
            <index> i </index>
            <value> z </value>
        </element>
    </constraints>
    <quantification>
        <exists>
            <vars> x </vars>
            <minimize> s </minimize>
        </exists>
        <forall>
            <vars> i </vars>
            <ctrs> res </ctrs>
            <aggregate id="s"> sum(z) </aggregate>
        </forall>
        <exists>
            <vars> z </vars>
        </exists>
    </quantification>
</instance>
```
11.5 Stochastic Constraint Reasoning

Two frameworks have been introduced for dealing with uncontrollable variables: SCSP (Stochastic Constraint Satisfaction Problem) and SCOP (Stochastic Constraint Optimization Problem).

11.5.1 SCSP

To define a SCSP instance, in XCSP3, you have to:

- set the attribute type of <instance> to "SCSP";
- enumerate variables (at least one) within <variables>;
- enumerate constraints (at least one) within <constraints>; either there is an attribute threshold associated with <constraints> or an attribute threshold associated with each constraint element present in <constraints>.
- handle stages with an element <stages>, which alternately contains elements <decision> and <stochastic>; in each of these elements, we find a sequence of variable ids.

The syntax is:

```
<instance format="XCSP3" type="SCSP">
  <variables>
    (<var.../> | <array.../>)+
  </variables>
  <constraints [threshold="number"]>
    (<constraint [threshold="number"].../> | <group.../> | <block.../>)*
  </constraints>
  <stages>
    (<decision> (intVar wspace)+ <decision> | <stochastic> (intVar wspace)+ <stochastic>+</stochastic>)*
  </stages>
  [<annotations.../>]
</instance>
```

Here is an illustration taken from [141] about modeling a simple m quarter production planning problem. In each quarter, we will sell between 100 and 105 copies of a book. To keep customers happy, we want to satisfy demand in all m quarters with 80% probability. At the start of each quarter, we decide how many books to print for that quarter. There are m decision variables, \( x_i \) representing production in each quarter. There are also m stochastic variables, \( y_i \) representing demand in each quarter. These take values between 100 and 105 with equal probability. There is a constraint to ensure 1st quarter production meets 1st quarter demand: \( x_1 \geq y_1 \). There is also a constraint to ensure 2nd quarter production meets 2nd quarter demand plus any unsatisfied demand or less any stock: \( x_2 \geq y_2 + (y_1 - x_1) \). And there is a constraint to ensure jth \((j \geq 2)\) quarter production meets jth quarter demand plus any unsatisfied demand or less any stock: \( x_j \geq y_j + \sum_{i=1}^{j-1} (y_i - x_i) \). We must satisfy these constraints with a threshold probability of 0.8.

For our illustration, with \( m = 2 \), we obtain:
Suppose now that there is a specific threshold 0.7 and 0.9 for the two constraints. The elements <constraints> become:

Example 175

```xml
<constraints>
  <intension threshold="0.7">
    ge(x1, y1)
  </intension>
  <intension threshold="0.9">
    ge(x2, add(y2, sub(y1, x1))
  </intension>
</constraints>
```

11.5.2 SCOP

To define a SCOP instance, in XCSP3, you have to:

- set the attribute type of <instance> to "SCOP";
- enumerate variables (at least one) within <variables>;
- enumerate constraints (at least one) within <constraints>;
- handle stages with an element <stages>;
- enumerate objectives (at least one) within <objectives>;

11.6 Qualitative Spatial Temporal Reasoning

Qualitative Spatial Temporal Reasoning (QSTR) deals with qualitative calculi. A qualitative calculus is defined from a finite set $B$ of base relations on a domain $D$. The elements of $D$
represent temporal or spatial entities, and the elements of \( B \) represent all possible configurations between two entities. \( B \) is a set that satisfies the following properties [87]: \( B \) forms a partition of \( D \times D \), \( B \) contains the identity relation \( \text{Id} \), and \( B \) is closed under the converse operation \( \leftarrow \). A (complex) relation is the union of some base relations, but it is customary to represent a relation as the set of base relations contained in it. Hence, the set \( 2^B \) represents the set of relations of the qualitative calculus.

A QSTR instance is a pair composed of a set of variables and a set of constraints. Each variable represents a spatial/temporal entity of the system that is modeled. Each constraint represents a set of acceptable qualitative configurations between two variables and is defined by a relation.

### 11.6.1 Interval Calculus

A well known temporal qualitative formalism is the Interval Algebra, also called Allen’s calculus [2]. The domain \( D_{\text{int}} \) of this calculus is the set \( \{(x^-, x^+) \in \mathbb{Q} \times \mathbb{Q} : x^- < x^+\} \) since temporal entities are intervals of the rational line. The set \( B_{\text{int}} \) of this calculus is the set \( \{\text{eq}, \text{p}, \text{i}, \text{m}, \text{mi}, \text{o}, \text{oi}, \text{s}, \text{si}, \text{d}, \text{di}, f, fi\} \) of thirteen binary relations representing all orderings of the four endpoints of two intervals. For example, \( m = \{(x^-, x^+), (y^-, y^+)\} \in D_{\text{int}} \times D_{\text{int}} : x^+ = y^- \).

To define a QSTR instance based on the Interval Algebra, in XCSP3, you have to:

- set the attribute type of \(<\text{instance}>\) to “QSTR”;
- enumerate interval variables (at least one) within \(<\text{variables}>\);
- enumerate interval constraints within \(<\text{constraints}>\).

The syntax is:

```
<instance format="XCSP3" type="QSTR">
  <variables>
    (#var type="interval".../> | <array type="interval".../>)+
  </variables>
  <constraints> <interval.../>* </constraints>
  [<annotations.../>]
</instance>
```

**Remark 52** Although not indicated above, groups and blocks of interval constraints can also be used.
As seen in Chapter 6, an interval constraint is defined by an element `<interval>` that contains an element `<scope>` and an element `<relation>`. An illustration is given for the qualitative constraint network depicted above. Its XCSP3 encoding is given by:

```
Example 176
<instance format="XCSP3" type="QSTR">
  <variables>
    <var id="v0" type="interval"/>
    <var id="v1" type="interval"/>
    <var id="v2" type="interval"/>
    <var id="v3" type="interval"/>
  </variables>
  <constraints>
    <interval>
      <scope> v0 v1 </scope>
      <relation> di m s </relation>
    </interval>
    <interval>
      <scope> v0 v3 </scope>
      <relation> m s fi </relation>
    </interval>
    <interval>
      <scope> v1 v2 </scope>
      <relation> o oi </relation>
    </interval>
    <interval>
      <scope> v2 v3 </scope>
      <relation> d o fi </relation>
    </interval>
  </constraints>
</instance>
```

### 11.6.2 Point Calculus

Point Algebra (PA) is a simple qualitative algebra defined for time points.

To define a QSTR instance based on the Point Algebra, in XCSP3, you have to:
• set the attribute type of <instance> to "QSTR",
• enumerate point variables (at least one) within <variables>;
• enumerate point constraints within <constraints>.

The syntax is:

**Syntax 177**

```
<instance format="XCSP3" type="QSTR">
  <variables>
    (<var type="point"/> | <array type="point"/>)+
  </variables>
  <constraints>
    <point/>
  </constraints>
</instance>
```

**Remark 53** Although not indicated above, groups and blocks of point constraints can also be used.

As seen in Chapter 6, a point constraint is defined by an element <point> that contains an element <scope> and an element <relation>. An illustration is given by:

**Example 177**

```
<instance format="XCSP3" type="QSTR">
  <variables>
    <var id="x" type="point"/>
    <var id="y" type="point"/>
    <var id="z" type="point"/>
  </variables>
  <constraints>
    <point>
      <scope> x y </scope>
      <relation> b a </relation>
    </point>
    <point>
      <scope> x z </scope>
      <relation> b eq </relation>
    </point>
    <point>
      <scope> y z </scope>
      <relation> a </relation>
    </point>
  </constraints>
</instance>
```

### 11.6.3 Region Connection Calculus

RCC8 consists of 8 basic relations that are possible between two regions:
- disconnected (DC)
- externally connected (EC)
- equal (EQ)
- partially overlapping (PO)
- tangential proper part (TPP)
- tangential proper part inverse (TPPi)
- non-tangential proper part (NTPP)
- non-tangential proper part inverse (NTPPi)

So, as before, to define a QSTR instance based on the Region Connection Calculus 8, in XCSP3, you have to:
- set the attribute type of <instance> to "QSTR",
- enumerate region variables (at least one) within <variables>;
- enumerate rcc8 constraints within <constraints>.

As seen in Chapter 6, a rcc8 constraint is defined by an element <region> with attribute type set to "rcc8" and containing an element <scope> and an element <relation>.

### 11.6.4 TCSP

A well-known framework for time reasoning is TCSP (Temporal Constraint Satisfaction Problem) [43]. In this framework, variables represent time points and temporal information is represented by a set of unary and binary constraints, each specifying a set of permitted intervals.

For this framework, as seen in Chapter 6, we only need to introduce a type of constraints, called dbd constraints, for disjunctive binary difference constraints (as in [72]). A unary dbd constraint on variable $x$ has the form:

$$a_1 \leq x \leq b_1 \lor \ldots \lor a_p \leq x \leq b_p$$

A binary dbd constraint on variables $x$ and $y$ has the form:

$$a_1 \leq y - x \leq b_1 \lor \ldots \lor a_p \leq y - x \leq b_p$$

where $x$ and $y$ are real variables representing time points and $a_1, \ldots, a_p, b_1, \ldots, b_p$ are real numbers denoting the bounds of $p$ considered intervals.

In XCSP3, a dbd constraint is defined by an element <dbd> that contains an element <scope> and an element <intervals>. When a scope contains two variables $x, y$ in that order, it means that the difference is $y - x$.

To define a TCSP instance, in XCSP3, you have to:
- set the attribute type of <instance> to "TCSP",
- enumerate point variables (at least one) within <variables>;
- enumerate dbd constraints within <constraints>.

The syntax is:
Here is an example taken from [43]. John goes to work either by car (30-40 minutes) or by bus (at least 60 minutes). Fred goes to work either by car (20-30 minutes) or in a car pool (40-50 minutes). Today John left home between 7:10 and 7:20, and Fred arrived at work between 8:00 and 8:10. We also know that John arrived at work about 10-20 minutes after Fred left home.

Following [72], we can introduce the following variables: $x_0$ for a special time point denoting the beginning of time (7:00 in our case), $x_1$ for the time at which John left home, $x_2$ for the time at which John arrived at work, $x_3$ for the time at which Fred left home, and $x_4$ for the time at which Fred arrived at work. We then obtain:

```
<instance format="XCSP3" type="TCSP">
<variables>
  <var id="x0" type="point"/>
  <var id="x1" type="point"/>
  <var id="x2" type="point"/>
  <var id="x3" type="point"/>
  <var id="x4" type="point"/>
</variables>
<constraints>
  <dbd>
    <scope> x0 x1 </scope>
    <intervals> [10,20] </intervals>
  </dbd>
  <dbd>
    <scope> x1 x2 </scope>
    <intervals> [30,40] [60,+infinity[ </intervals>
  </dbd>
  <dbd>
    <scope> x3 x4 </scope>
    <intervals> [20,30] [40,50] </intervals>
  </dbd>
  <dbd>
    <scope> x3 x2 </scope>
    <intervals> [10,20] </intervals>
  </dbd>
  <dbd>
    <scope> x0 x4 </scope>
    <intervals> [60,70] </intervals>
  </dbd>
</instance>
```
11.7 Continuous Constraint Solving

11.7.1 NCSP

Numerical CSP instances are defined as CSP instances, but involve real variables and constraints.

To define a NCSP instance, in XCSP3, you have to:

- set the attribute type of <instance> to "NCSP";
- enumerate variables (at least one) within <variables>;
- enumerate constraints within <constraints>;

The syntax is as follows:

**Syntax 179**

```xml
<instance format="XCSP3" type="NCSP">
  <variables>
    >/var type="real".../> | <array type="real".../>+</variables>
  <constraints> <intension.../> | <sum.../>* </constraints>
</instance>
```

**Example 179**

```xml
<instance format="XCSP3" type="NCSP">
  <variables>
    <var id="x" type="real">[-4,4]</var>
    <var id="y" type="real">[-4,4]</var>
  </variables>
  <constraints>
    <intension> eq(sub(y,pow(x,2)),0) </intension>
    <intension> eq(sub(y,add(x,1)),0) </intension>
  </constraints>
</instance>
```

11.7.2 NCOP

Numerical COP instances are defined as COP instances, but involve real variables and constraints.

To define a NCOP instance, in XCSP3, you have to:

- set the attribute type of <instance> to "NCOP";
- enumerate variables (at least one) within <variables>;

```xml
<instance format="XCSP3" type="NCSP">
  <variables>
    </var type="real".../> | <array type="real".../>+</variables>
  <constraints> <intension.../> | <sum.../>* </constraints>
</instance>
```
• enumerate constraints within `<constraints>`;
• enumerate objectives (at least one) within `<objectives>`.

The syntax is as follows:

```
Syntax 180
<instance format="XCSP3" type="NCOP">
  <variables>
    ( <var type="real".../> | <array type="real".../> )*  
  </variables>
  <constraints> ( <intension.../> | <sum.../> )* </constraints>
  <objectives [combination="combinationType"]>
    ( <minimize.../> | <maximize.../> )*  
  </objectives>
  [ <annotations.../> ]
</instance>
```

11.8 Distributed Constraint Reasoning

Distributed Constraint Reasoning has been studied for both satisfaction and optimization.

11.8.1 DisCSP

DisCSP (Distributed CSP) instance, is defined as a classical CN \( P = (\mathcal{X}, \mathcal{C}) \), together with a set of \( p \) agents \( \mathcal{A} = \{a_1, a_2, \ldots, a_p\} \). Each agent \( a \in \mathcal{A} \) controls a proper subset of variables \( \text{vars}(a) \) of \( \mathcal{X} \), and knows a subset of constraints \( \text{ctrs}(a) \) of \( \mathcal{C} \). The sets of controlled variables of all agents form a partition of \( \mathcal{X} \). When an agent \( a_i \) can directly send messages to another agent \( a_j \), \( a_j \) is said to be reachable from \( a_i \).

To describe the way agents are defined and interact, we add to `<instance>` an element `<agents>` that contains a sequence of elements `<agent>`. Each element `<agent>` has an optional attribute `id` and contains an element `<vars>`, an element `<ctrs>` and a sequence of elements `<comm>`. Note that, as described below, some of these intern elements can be omitted in certain cases. They are defined as follows:

- The element `<vars>` contains the list of variables, given by their ids, controlled by the agent.
- The element `<ctrs>` contains the list of constraints, given by their ids, known by the agent.
- When there is no (special) communication cost between agents, there is only one element `<comm>` that contains the list of agents, given by their ids, reachable from the agent.
- When there are specific communication costs between agents, there is a sequence of elements `<comm>`, each one that contains the list of agents, given by their ids, reachable from the agent with a cost given by an attribute `cost`.

Note that the introduced elements `<comm>`, taken together, allows us to describe the topology of the communication graph/network.

There are special cases where some elements can be omitted. It depends on the general configuration of agents, which is characterized by three (optional) attributes associated with
the element `<agents>`. The optional attribute `varsModel` characterizes the partition of variables between agents. When there is a bijection between $X$ and $A$, i.e., each agent controls exactly one distinct variable, the value of `varsModel` is set to "bijection". The optional attribute `ctrsModel` characterizes the knowledge of agents on constraints. When every agent $a$ exactly knows the set of constraints involving at least one variable controlled by the agent, i.e., $ctrs(a) = \{ c \in C \mid scp(c) \cap vars(a) \neq \emptyset \}$, the value of `ctrsModel` is set to "TKC", which stands for Totally Known Constraints. The optional attribute `commModel` characterizes the way agents can communicate, i.e. what is the topology of the communication network. When the communication graph is the same as the constraint (primal) graph, the value of `commModel` is set to "scope".

Now, here are the possible simplifications according to the value of these three attributes. First, for any element `<agent>`, the sequence of elements `<comm>` can be empty when there is no (special) communication cost between agents and when the value of the attribute `commModel` is "scope", because it is possible to infer the communication graph in that case. Similarly, for any element `<agent>`, the element `<ctrs>` can be omitted when the value of `ctrsModel` is "TKC". Finally, when `varsModel` has value "bijection", `ctrsModel` has value "TKC" and `commModel` has value "scope", we can avoid to introduce all intern elements `<agent>`.

To define a DisCSP instance, in XCSP3, you have to:

- set the attribute `type` of `<instance>` to "DisCSP",
- enumerate variables (at least one) within `<variables>`,
- enumerate constraints (at least one) within `<constraints>`;
- enumerate agents within `<agents>`.

An illustration is given below:

```xml
<instance format="XCSP3" type="DisCSP">
  <variables>
    <var id="w"> red green blue </var>
    <var id="x"> red green blue </var>
    <var id="y"> red green blue </var>
    <var id="z"> red green blue </var>
  </variables>
  <constraints>
    <intension id="c1"> ne(w,x) </intension>
    <intension id="c2"> ne(w,y) </intension>
    <intension id="c3"> ne(w,z) </intension>
    <intension id="c4"> ne(x,z) </intension>
    <intension id="c5"> ne(y,z) </intension>
  </constraints>
  <agents>
    <agent id="a1">
      <vars> w </vars>
      <ctrs> c1 c2 c3 </ctrs>
      <comm> a2 a3 a4 </comm>
    </agent>
    <agent id="a2">
      <vars> x </vars>
    </agent>
  </agents>
</instance>
```
Interestingly enough, for the element `<agents>`, we can use the following abridged version:

```
Example 181

<agents varsModel="bijection"
        ctrsModel="TKC"
        commModel="scope" />
```

Let us suppose now that we only consider three agents, one controlling the variables $w$ and $x$, and the two others controlling the variables $y$ and $z$, respectively. We keep a TKC model, and consider that the communication cost between any pair of agents is 10, except between the first and third agents, where it is 100. We then obtain the following element `<agents>`:

```
Example 182

<agents ctrsModel="TKC" commModel="scope">
    <agent id="a1">
        <vars> w x </vars>
        <ctrs> c1 c2 c3 c4 </ctrs>
        <comm cost="10"> a2 </comm>
        <comm cost="100"> a3 </comm>
    </agent>
    <agent id="a2">
        <vars> y </vars>
        <ctrs> c2 c5 </ctrs>
        <comm cost="10"> a1 a3 </comm>
    </agent>
    <agent id="a3">
        <vars> z </vars>
        <ctrs> c3 c4 c5 </ctrs>
        <comm cost="100"> a1 </comm>
        <comm cost="10"> a2 </comm>
    </agent>
</agents>
```
Because the model is TKC, we could discard all elements $<\text{ctrs}>$.

### 11.8.2 DisWCSP (DCOP)

A DisWCSP (Distributed WCSP) instance, often called DCOP in the literature, is defined as a classical WCN $P = (\mathcal{X}, \mathcal{C}, k)$, together with a set of $p$ agents $\mathcal{A} = \{a_1, a_2, \ldots, a_p\}$. The way the agents are defined are exactly as for DisCSP.

To define a DisWCSP instance, in XCSP3, you have to:
- set the attribute type of $<\text{instance}>$ to "DisWCSP",
- enumerate variables (at least one) within $<\text{variables}>$;
- enumerate weighted constraints (at least one) within $<\text{constraints}>$;
- enumerate agents within $<\text{agents}>$. 
In XCSP3, it is possible to insert an element `<annotations>` to express search and filtering advice. In this chapter, we present this while considering integer variables only (i.e., CSP and COP frameworks), but of course, what follows can be directly adapted to other frameworks.

Recall that, in XCSP3, an `intVar` corresponds to the id of a variable declared in `<variables>`. Similarly, an `intCtr` corresponds to the id of a constraint declared in `<constraints>`. Besides, all BNF non-terminals such as `filteringType`, `consistencyType`, `branchingType` and `restartsType` are defined in Appendix B. The general syntax for `<annotations>` is as follows:

```
Syntax 181
<annotations>
  [\<decision\> (intVar wspace)+ \</decision\>]
  [\<output\> (intVar wspace)+ \</output\>]
  [\<varHeuristic.../>]
  [\<valHeuristic.../>]
  (\<filtering type="filteringType"> (intCtr wspace)+ \</filtering\>)*
  [\<filtering type="filteringType" />]
  [\<prepro consistency="consistencyType" />]
  [\<search [consistency="consistencyType"] [branching="branchingType"] />]
  [\<restarts type="restartsType" cutoff="unsignedInteger" [factor="decimal"] />]
</annotations>
```

### 12.1 Annotations about Variables

It is possible to associate annotations with variables by introducing a few elements inside `<annotations>`. First, one can indicate the variables that the solver should branch in, so-called branching or decision variables, and the variables that are considered relevant when outputting solutions. This is made possible by introducing an element `<decision>` and an
element <output> inside <annotations>. For example, below, we identify variables $x_0$, $x_1$ and $x_2$ as being three decision variables, and variables $y$ and $z$ as being those that should be displayed whenever a solution is found.

Example 183

```
<annotations>
  <decision> $x_0$ $x_1$ $x_2$ </decision>
  <output> $y$ $z$ </output>
</annotations>
```

It may be interesting to indicate which heuristic(s) should be followed by a solver. This is the role of the elements <varHeuristic> and <valHeuristic>. For variable ordering, one can introduce several elements inside <varHeuristic>, chosen among <static>, <random>, <min> and <max>. An element <static> indicates in which exact order variables contained in it must be selected by the solver. An element <random> indicates that variables contained in it should be selected at random. An element <min> or <max> has a required attribute type whose value indicates the type of variable ordering heuristic, based on one or several criteria, that must be followed for the variables it contains. Basic types, based on elementary criteria, are currently:

- "lexico".
- "dom" [64].
- "deg" [131], "ddeg" and "wdeg" [26]
- "impact" [58, 111]
- "activity" [94]

It is possible to combine such criteria with the operator "/", thus considering ratios, and also with the operator "+" to determine how to break ties.

We can then use complex types, as for example:

- "dom/deg", "dom/ddeg" and "dom/wdeg"
- "dom+ddeg"

For example, "dom+ddeg" is known as the Brelaz heuristic [27, 128].

Note that the BNF syntax for all possible values of type is given by varhType in Appendix B. The syntax for <varHeuristic> is then:

```
Syntax 182
<varHeuristic [lc="unsignedInteger"]>
  ( <static> (intVar wspace)+ </static>
   | <random> (intVar wspace)+ </random>
   | <min type="varhType"> (intVar wspace)+ </min>
   | <max type="varhType"> (intVar wspace)+ </max>
 )*
[<random/> | <min type="varhType"/> | <max type="varhType"/>] // default
</varHeuristic>
```

As you can see, there may be several elements inside <varHeuristic>, including one in last position that may contain nothing. Actually, when there are several elements inside
<varHeuristic>, it means that the variables of the first element should be selected first (using the semantics of the element), then the variables of the second element should be selected, and so on. Finally, when there is an element at last position without any content, it means that it applies to all remaining variables (i.e., those not explicitly listed in previous elements): this is the default heuristic. By means of the attribute \( lc \), it is also possible to indicate that last conflict reasoning [81] should be employed: the value of the attribute indicates the maximal size \( (k) \) of the testing set.

If the advice is simply to use \( \text{min dom/wdeg} \) over the entire constraint network, you will write:

**Example 184**

```xml
<varHeuristic>
  <min type="dom/wdeg"/>
</varHeuristic>
```

In another example, below, we assume that the solver should select variables \( x_0, x_7 \) and \( x_{12} \) first (in that order), then variables \( x_{10}, x_{20} \) in an order that depends on their domain sizes, and finally all other variables by considering at each decision step the variable with the greatest weighted degree. On our example, it is also advised to use \( lc(2) \).

**Example 185**

```xml
<varHeuristic lc="2">
  <static> x0 x7 x12 </static>
  <min type="dom"> x10 x20 </min>
  <max type="wdeg"/>
</varHeuristic>
```

For value ordering, one can introduce several elements inside <valHeuristic>, chosen among <static>, <random>, <min> and <max>. An element <static> indicates in which order values for the variables contained in it should be selected by the solver. The order for values is given by the attribute order. An element <random> indicates that values for the variables contained in it should be selected randomly. An element <min> or <max> has a required attribute type whose value indicates the type of value ordering heuristic that must be followed for the variables it contains. Currently, possible values of type (i.e., values of valhType defined in Appendix B) are:

- "conflicts"
- "value".

Here, conflicts refers to the number of conflicts with neighbors [95, 57].

The syntax for <valHeuristic> is then:

**Syntax 183**

```xml
<valHeuristic>
  ( <static order="(intVal wspace)+"> (intVar wspace)+ </static>
   | <random> (intVar wspace)+ </random>
   | <min type="valhType"> (intVar wspace)+ </min>
   | <max type="valhType"> (intVar wspace)+ </max>
</valHeuristic>
```
If the advice is simply to use min value over the entire constraint network, you will write:

Example 186

```xml
<valHeuristic>
  <min type="value"/>
</valHeuristic>
```

Another example is given below. Whenever \( x \) is selected, the solver must choose the first valid value in the order given by order. Whenever \( y \) and \( z \) are selected, the solver must select the value that has the smallest number of conflicts (with its neighbors). Finally, whenever any other variable is selected by the solver, the greatest value in the domain of the selected variable must be chosen.

Example 187

```xml
<valHeuristic>
  <static order = "4 5 3 6 2 1"> x </static>
  <min type="conflicts"> y z </min>
  <max type="value"/>
</valHeuristic>
```

12.2 Annotations about Constraints

It is possible to associate annotations with constraints by introducing a few elements inside <annotations>. First, one can indicate the level of filtering that the solver should try to enforce on specified constraints by using elements <filtering> with the type attribute set to the wished value. Currently, there are five possible values for type:

- "boundsZ" for integer bounds propagation. See Bounds(Z) in [39].
- "boundsD" for a stronger integer bounds propagation. See Bounds(D) in [39].
- "boundsR" for real bounds propagation. See Bounds(R) in [39].
- "AC" for (Generalized) Arc Consistency. This value is valid for both binary and non-binary constraints (when it is usually called GAC).
- "FC" for Forward Checking. In that case, the element <filtering> has an optional attribute delay that indicates the number of unassigned variables under which AC should be enforced (since FC is a partial form of AC).

When an element <filtering> contains nothing, it must be the last element, meaning that it applies to all remaining constraints (i.e., those not explicitly listed in previous elements).

An example is given below. Here, the solver should enforce Bounds(Z) on constraints \( c_0 \), \( c_1 \), and \( c_2 \). Bounds(D) on constraints \( c_3 \) and \( c_4 \) and full (generalized) arc consistency on all other constraints.
12.3 Annotations about Preprocessing and Search

In some cases, it may be useful to indicate the overall level of consistency that the solve should enforce at preprocessing (i.e., before search) and/or during search; for this purpose, we introduce elements <prepro> and <search>. Currently, this level of consistency can be any value among:

- "FC" for Forward Checking
- "BC" for Bounds Consistency
- "AC" for (Generalized) Arc Consistency (i.e., for both binary and non-binary constraints)
- "SAC" for Singleton Arc Consistency
- "FPWC" for Full Pairwise Consistency
- "PC" for Path Consistency
- "CDC" for Conservative Dual Consistency
- "FDAC" for Full Directional Arc Consistency
- "EDAC" for Existential Directional Arc Consistency
- "VAC" for Virtual Arc Consistency

To record this information, we use an attribute consistency whose value is one among those cited just above. For search, we can also indicate the kind of branching that the solver should use, by means of the attribute branching whose value must currently be one among "2-way" and "d-way". Finally, we can give relevant information about restarts. We just have to introduce an element <restarts> with two attributes type and cutoff. Currently, possible values for type are:

- "luby"
- "geometric"

The attribute cutoff indicates the number of conflicts that must be encountered before restarting the first time. When the progression is geometric, there is an additional attribute factor.

An example is given below.

Example 188

```xml
<annotations>
  <filtering type="boundsZ"> c0 c1 c2 </filtering>
  <filtering type="boundsD"> c3 c4 </filtering>
  <filtering type="AC"/>
</annotations>
```

Example 189

```xml
<prepro 
  consistency="SAC"/>
<search 
  consistency="AC" 
  branching="2-way" />
<restarts 
  type="geometric" 
  cutoff="10" 
  factor="1.1"/>
```

Acknowledgments

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Appendices
XCSP3 keywords are:

neg, abs, add, sub, mul, div, mod, sqr, pow, min, max, dist, lt, le, ge, gt, ne, eq, set,
in, not, and, or, xor, iff, imp, if, card, union, inter, diff, sdiff, hull, djoint, subset,
subseq, supseq, supset, convex, PI, E, fdiv, fmod, sqrt, nroot, exp, ln, log, sin, cos,
tan, asin, acos, atan, sinh, cosh, tanh, others

There are some restrictions about the identifiers that can be used in XCSP3:

- the set of keywords, the set of symbolic values and the set of id values (i.e., values of attributes id) must all be disjoint,
- it is not permitted to have two attributes id with the same value,
- it is not permitted to have an attribute id of an element <array> that corresponds to a prefix of any other attribute id,
- it is not permitted to have an attribute id of an element <group> that corresponds to a prefix of any other attribute id.

Recall that characters “[“ and “]” are not allowed in identifiers. If you need to build a subnetwork (new file) with a selection of variables (from arrays) and constraints (from groups), it is recommended to adopt the following usage: replace each occurrence of the form “[i]” by “_i”. For example, x[0][3] becomes x_0_3 and g[0] becomes g_0.
The syntax given in this document combines two languages:

- XML for describing the main elements and attributes of XCSP3 (high level description)
- BNF for describing the textual contents of XCSP3 elements and attributes (low level description)

At some places, we need to postpone the description of some XML elements. In that case, we just employ `<elt.../>` to stand for an XML element of name `elt`, whose description is given elsewhere. This roughly corresponds to the notion of XML non-terminal. For example, because, as seen below, `|`, parentheses and `*` respectively stand for alternation, grouping and repetition (at least one), for indicating that the element `<variables>` can contain some elements `<var>` and `<array>`, we just write:

```
<variables>
  (<var.../> | <array.../>)*
</variables>
```

The “XML non-terminals” `<var.../>` and `<array.../>` must then be precisely described at another place. For example, for `<var.../>`, we can use the following piece of XCSP3 syntax, where one can observe that we have an XML element `<var>` with two attributes `id` and `type` (optional). The value of the attribute `id` as well as the content of the element `<var>` is defined using BNF, with BNF non-terminals written in dark blue italic form. The non-terminals, referred to all along the document, are defined in this section.

```
<var id="identifier" [type="integer"]>
  ((intVal | intIntvl) wspace)*
</var>
```

In many situations (as for our example above), the content of an XML text-only element corresponds to a list of basic data (values, variables, ...) with whitespace as separator. In such situations, the whitespace that follows the last object of the list must always be considered as optional.
More generally, the following rule applies for XCSP3:

Remark 54 In XCSP3, leading and trailing whitespace are tolerated, but not required, in any XML text-only element.

We use a variant of Backus-Naur Form (BNF) defined as follows:

- a non-terminal definition is preceded by ::= 
- terminals are between quotation marks as e.g., "CSP"
- non-terminals are composed of alphanumeric characters (and the character "_"), and so, do not contain any white space character, as e.g., frameworkType. When such non-terminals are mixed with XML notation (in the document), they are written in dark blue italic form as e.g., frameworkType
- concatenation is given by any non-empty sequence of white space characters
- alternatives are separated by a vertical bar as, e.g., "+ | "-"
- square brackets are used for surrounding optional elements, as, e.g., ["+ | "-"]
- an element followed by * can occur 0 or any number of times
- an element followed by + must occur at least 1 time
- an element followed by n+ must occur at least n times
- parentheses are used for grouping (often used with *, + and n+)

The syntax is given for fully expanded XCSP3 code, meaning that compact lists of array variables (such as $x[]$) are not handled.

The basic BNF non-terminals used in this document are:

```plaintext
wspace ::= (" " | \t | \n | \r)+
digit ::= "0".."9"
unsignedInteger ::= digit+
integer ::= ["+" | "]-"] unsignedInteger
decimal ::= integer "." unsignedInteger
rational ::= integer "/" unsignedInteger
number ::= integer | decimal | rational

boolean ::= "false" | "true"
intSet ::= "set(" [integer ("," integer)*] ")"

letter ::= "a".."z" | "A".."Z"
identifier ::= letter (letter | digit | "]" )*"`

The basic data type are (note that we include many aliases so as to facilitate readability):

indexing ::= ("[" unsignedInteger "]")+
variable ::= identifier [indexing]

01Var ::= variable
intVar ::= variable
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symVar ::= variable
realVar ::= variable
setVar ::= variable
graphVar ::= variable
qualVar ::= variable

intval ::= integer
realval ::= number
intIntvl ::= (integer | "-infinity") ".." (integer | "+infinity")
realIntvl ::= ("[" | "]" (number | "-infinity") ","
             (number | "+infinity") "]" | "]")
setVal ::= "{" [integer ("," integer)*] "}"
proba ::= unsignedInteger "." unsignedInteger
          | unsignedInteger "/" unsignedInteger
          | 0 | 1
operator ::= "lt" | "le" | "ge" | "gt" | "ne" | "eq" | "in" | "notin"
operand ::= intval | intVar | intIntvl | intSet

intvalShort ::= intval | "*"
intvalCompressed ::= intval | "*" | setVal

state ::= identifier
symbol ::= identifier
intCtr ::= identifier
dimensions ::= indexing

Some types are defined as:

frameworkType ::=
"CSP" | "COP" | "WCSP" | "FCSP" | "QCSP" | "QCSP+" | "QCOP" | "QCOP+
| "SCSP" | "SCOP" | "QSTR" | "TCSmP"
| "NCSP" | "NCOP" | "DisCSP" | "DisWCSP"

varType ::= "integer" | "symbolic" | "real" | "set" | "symbolic set"
         | "undirected graph" | "directed graph" | "stochastic"
         | "symbolic stochastic" | "point" | "interval" | "region"

orderedType ::= "increasing" | "strictlyIncreasing"
              | "decreasing" | "strictlyDecreasing"

rankType ::= "any" | "first" | "last"

iaBaseRelation ::= "eq" | "p" | "pi" | "m" | "mi" | "o" | "oi"
paBaseRelation ::= "b" | "eq" | "a"

rcc8BaseRelation ::= "dc" | "ec" | "eq" | "po" |
| "tpp" | "tppi" | "nttp" | "nttpi"

measureType ::= "var" | "dec" | "val" | "edit"

combinationType ::= "lexico" | "pareto"

blockType ::= "clues" | "symmetryBreaking" | "redundantConstraints" | "nogoods"

varhType ::= "lexico" | "dom" | "deg" | "ddeg" | "wdeg" | "immpact" | "activity"
| varhType="/" varhType
| varhType="/" varhType

valhType ::= "conflicts" | "value"

filteringType ::= "boundsZ" | "boundsD" | "boundsR" | "AC" | "FC"

consistencyType ::= "FC" | "BC" | "AC" | "SAC" | "FPWC" | "PC" | "CDC" | "FDAC" | "EDAC" | "VAC"

branchingType ::= "2-way" | "d-way"

restartsType ::= "luby" | "geometric"

The grammar used to build predicate expressions with integer variables is:

```
intExpr ::= boolean | integer
| "neg"( intExpr )"
| "abs"( intExpr )"
| "add"( intExpr ("," intExpr)+ )"
| "sub"( intExpr ("," intExpr )"
| "mul"( intExpr ("," intExpr )"
| "div"( intExpr ("," intExpr )"
| "mod"( intExpr ("," intExpr )"
| "sqr"( intExpr )"
| "pow"( intExpr ("," intExpr )"
| "min"( intExpr ("," intExpr )"
| "max"( intExpr ("," intExpr )"
| "dist"( intExpr ("," intExpr )"
| "if"( boolean ("," intExpr )"

| "f" | "fi"

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boolExpr ::= 
   01Var
   | "lt(" intExpr "," intExpr ")"
   | "le(" intExpr "," intExpr ")"
   | "ge(" intExpr "," intExpr ")"
   | "gt(" intExpr "," intExpr ")"
   | "ne(" intExpr "," intExpr ")"
   | "eq(" intExpr ("," intExpr)+ ")"
   | "in(" intExpr "," intSet ")"
   | "not(" boolExpr ")"
   | "and(" boolExpr ("," boolExpr)+ ")"
   | "or(" boolExpr ("," boolExpr)+ ")"
   | "xor(" boolExpr ("," boolExpr)+ ")"
   | "iff(" boolExpr ("," boolExpr)+ ")"
   | "imp(" boolExpr "," boolExpr ")"

smartExpr ::= 
   "lt(" intVar "," (intVar | intVal) ")"
   | "le(" intVar "," (intVar | intVal) ")"
   | "ge(" intVar "," (intVar | intVal) ")"
   | "gt(" intVar "," (intVar | intVal) ")"
   | "ne(" intVar "," (intVar | intVal) ")"
   | "eq(" intVar "," (intVar | intVal) ")"
   | "lt(" intVar ",add(" intVar "," intVal ")")"
   | "le(" intVar ",add(" intVar "," intVal ")")"
   | "ge(" intVar ",add(" intVar "," intVal ")")"
   | "gt(" intVar ",add(" intVar "," intVal ")")"
   | "ne(" intVar ",add(" intVar "," intVal ")")"
   | "eq(" intVar ",add(" intVar "," intVal ")")"
   | "in(" intVar "," intSet ")"
   | "not(in(" intVar "," intSet ")")"

The grammar used to build predicate expressions with integer variables and set variables is:

intExprSet ::= 
   01Var | intVar | integer
   | "neg(" intExprSet ")"
   | "abs(" intExprSet ")"
   | "add(" intExprSet "," intExprSet)+ ")"
   | "sub(" intExprSet "," intExprSet ")"
   | "mul(" intExprSet "," intExprSet)+ ")"
   | "div(" intExprSet "," intExprSet ")"
   | "mod(" intExprSet "," intExprSet ")"
   | "sqr(" intExprSet ")"
   | "pow(" intExprSet "," intExprSet ")"
   | "min(" intExprSet "," intExprSet)+ ")"
   | "max(" intExprSet "," intExprSet)+ ")"
   | "dist(" intExprSet "," intExprSet ")"
The grammar used to build predicate expressions with integer variables and real variables is:

realExpr ::=  
  01Var | intVar | realVar | number | PI | E  
  | "neg(" realExpr ")"  
  | "abs(" realExpr ")"  
  | "add(" realExpr "," realExpr)+ ")"  
  | "sub(" realExpr "," realExpr ")"  
  | "mul(" realExpr "," realExpr)+ ")"  
  | "fdiv(" realExpr "," realExpr ")"  
  | "fmod(" realExpr "," realExpr ")"  

setExpr ::=  
  setVar | "set(" [intExprSet "," intExprSet]* ")"  
  | "union(" setExpr "," setExpr)+ ")"  
  | "inter(" setExpr "," setExpr)+ ")"  
  | "diff(" setExpr "," setExpr ")"  
  | "sdiff(" setExpr "," setExpr)+ ")"  
  | "hull(" setExpr ")"  

boolExprSet ::=  
  01Var  
  | "lt(" intExprSet "," intExprSet ")"  
  | "le(" intExprSet "," intExprSet ")"  
  | "ge(" intExprSet "," intExprSet ")"  
  | "gt(" intExprSet "," intExprSet ")"  
  | "ne(" intExprSet "," intExprSet ")"  
  | "eq(" intExprSet "," intExprSet)+ ")"  
  | "in(" intExprSet "," setExpr ")"  
  | "ne(" setExpr "," setExpr ")"  
  | "eq(" setExpr "," setExpr)+ ")"  
  | "not(" boolExprSet ")"  
  | "and(" boolExprSet "," boolExprSet)+ ")"  
  | "or(" boolExprSet "," boolExprSet)+ ")"  
  | "xor(" boolExprSet "," boolExprSet)+ ")"  
  | "iff(" boolExprSet "," boolExprSet)+ ")"  
  | "imp(" boolExprSet "," boolExprSet ")"  
  | "djoint(" setExpr "," setExpr ")"  
  | "subset(" setExpr "," setExpr ")"  
  | "subseq(" setExpr "," setExpr ")"  
  | "supseq(" setExpr "," setExpr ")"  
  | "supset(" setExpr "," setExpr ")"  
  | "convex(" setExpr ")"
| "min(" realExpr ("," realExpr)+ ")" |
| "max(" realExpr ("," realExpr)+ ")" |
| "sqr(" realExpr ")" |
| "pow(" realExpr "," realExpr ")" |
| "sqrt(" realExpr ")" |
| "nroot(" realExpr "," integer ")" |
| "exp(" realExpr ")" |
| "ln(" realExpr ")" |
| "log(" realExpr "," integer ")" |
| "sin(" realExpr ")" |
| "cos(" realExpr ")" |
| "tan(" realExpr ")" |
| "asin(" realExpr ")" |
| "acos(" realExpr ")" |
| "atan(" realExpr ")" |
| "sinh(" realExpr ")" |
| "cosh(" realExpr ")" |
| "tanh(" realExpr ")" |
| "if(" boolExprReal "," realExpr "," realExpr ")" |

boolExprReal ::= 
    01Var 
    | "lt(" realExpr "," realExpr ")" 
    | "le(" realExpr "," realExpr ")" 
    | "ge(" realExpr "," realExpr ")" 
    | "gt(" realExpr "," realExpr ")" 
    | "ne(" realExpr "," realExpr ")" 
    | "eq(" realExpr ("," realExpr)+ ")" 
    | "not(" boolExprReal ")" 
    | "and(" boolExprReal ("," boolExprReal)+ ")" 
    | "or(" boolExprReal ("," boolExprReal)+ ")" 
    | "xor(" boolExprReal ("," boolExprReal)+ ")" 
    | "iff(" boolExprReal ("," boolExprReal)+ ")" 
    | "imp(" boolExprReal "," boolExprReal ")" |

Finally, we need some non-terminal tokens to be used for XML elements.

cost ::= 
    "extension" | "intension" | "smart" 
    | "regular" | "grammar" | "mdd" 
    | "allDifferent" | "allEqual" | "allDistant" | "ordered" | "allIncomparable" 
    | "sum" | "count" | "nValues" | "cardinality" | "balance" | "spread" | "deviation" | 
    | "sumCosts" | "sequence" 
    | "maximum" | "minimum" | "element" | "channel" | "permutation" | "precedence" 
    | "stretch" | "noOverlap" | "cumulative" | "binPacking" | "knapsack" | "networkFlow" 
    | "circuit" | "nCircuits" | "path" | "nPaths" | "tree" | "nTrees" 
    | "clause" | "instantiation" 
    | "allIntersecting" | "range" | "roots" | "partition" 
    | "arbo" | "nArbos" | "nCliques"
metaConstraint ::= "slide" | "seqbin" | "and" | "or" | "not" | "ifThen" | "ifThenElse"
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Note: See related documentation for "soft-slide" and "permutation\(\times\)increasing\" restrictions.
XML and JSON

JSON (JavaScript Object Notation) is another popular language-independent data format. We show in this appendix how we can translate XCSP3 instances from XML to JSON, without losing any information. However, note that although JSON allows us to build objects with duplicate keys, JSON decoders might handle those objects differently; we discuss this important issue at the end of this chapter. We basically adopt and adapt the rules proposed by O’Reilly (www.xml.com).

- Each XML attribute is represented as a name/value property. The name of the property is the name of the attribute preceded by @, and the value of the property is the value of the attribute.
- Each XML element is represented as a name/value property. The name of the property is the name of the element, and the value of the property is defined as follows:
  - If the element has no attribute and no content, the value is “null”.
  - If the element has no attribute and only a (non-empty) textual content, the value is the textual content.
  - In all other cases, the value is an object with each XML attribute and each child element represented as a property of the object. Note that a child text node is represented by a property with ”#text” as name and the text as value, except for the following special cases:
    * if the parent node name is “var” or “array”, then the name of the property (for the child text node) is “domain”,
    * if the parent node name is “intension”, then the name of the property (for the child text node) is “function”,
    * if the parent node name is “minimize” or “maximize”, then the name of the property (for the child text node) is either “expression” (if the objective has a functional form) or “list” (if the objective has a specialized form); see forms of objectives in Chapter 3,
* if the parent node name is “allDifferent”, “allEqual”, “ordered”, “channel”, “circuit”, “clause”, “cube”, or “allIntersecting”, the name of the property (for the child text node) is “list”.

- Refining the previous rule, any sequence of XML elements of same names, is represented as a property whose name is the name of the elements and the value an array containing the values of these elements in sequence.
- The order of elements must be preserved.
- The root element “instance” is not represented. We directly give the representation of the content of this element (together with its attributes). So, for a CSP instance, we obtain something like:

```json
{
  "@format": "XCSP3",
  "@type": "CSP",
  ...
}
```

instead of:

```json
{
  "instance": {
    "@format": "XCSP3",
    "@type": "CSP",
    ...
  }
}
```

An illustration of these rules (special cases are not illustrated) is given below:

```xml
<elt/>  "elt": null
<elt>  "elt": "text"
  text
</elt>
<elt att="val"  "elt": {
  />
<elt att1="val1" att2="val2"  "elt": {
  />
<elt att="val">  "elt": {
```

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An XSLT-based converter will be made available soon on our website.

**Remark 55** JSON is an attractive format. However, in our context of representing CP instances, the reader must be aware of two possible limitations, described below.

First, although it is theoretically valid to build objects with duplicate keys, JSON parsers may behave differently. For example,

```javascript
JSON.parse('{"a": "texta", "b": "textb", "a": "texta2"}')
```

yields with some parsers:

```javascript
{ a: "texta2", b: "textb" }
```

Just imagine what you can obtain with the following constraints:

```xml
<constraints>
  <intension> eq(x,0) </intension>
  <extension>
    <list> t[0] t[1] </list>
    <supports> (2,4)(3,5) </supports>
  </extension>
</constraints>
```
\text{intension} \ le(y,z) \ </ \text{intension} > \\
</ \text{constraints} >

translated in JSON as:

```
"constraints": {
  "intension": "eq(x,0)",
  "extension": {
    "list": "t[0] t[1]",
    "supports": "(2,4)(3,5)"
  },
  "intension": "le(y,z)"
}
```

If the parser you use has not the appropriate behavior for our purpose, you have to transform the JSON file, putting in arrays the contents of similar keys. In our example, this would give:

```
"constraints": {
  "intension": ["eq(x,0)", "le(y,z)"],
  "extension": {
    "list": "t[0] t[1]",
    "supports": "(2,4)(3,5)"
  }
}
```

Note that the initial order of constraints cannot be preserved. Besides, for some frameworks (e.g., SCSP, QCSP), it is not possible to proceed that way. We have to define other transformation rules for cases like:

```
<quantification>
  <exists> w x </exists>
  <forall> y </forall>
  <exists> z </exists>
</quantification>
```
• XCSP3, version 3.0.5. Published on November 25, 2017. The general introduction has been slightly modified; see Section 1.2. XCSP3-core is slightly modified: circuit enters XCSP3-core whereas stretch leaves XCSP3-core; see Section 1.5. The constraint ordered admits now an optional element specifying the minimum distance between two successive variables of the main list; see Section 4.1.3.4. The default value of the attribute closed used by the constraint cardinality is now "false", instead of "true", see Section 4.1.4.4. A useful variant of the constraint element, where a list of values is handled, has been added; see Section 4.1.5.3. A variant of the constraint channel, involving two lists of variables of different size, has been added; see Section 4.1.5.4. The meta-constraints ifThen and ifThenElse have been added; see Section 8.8 and 8.9. The constraint smart has been added; see Section 4.1.1.3. Forms of allDifferent and sum, related to the concept of views, are illustrated at the end of Section 4.1.3.1 and 4.1.4.1; these forms can be handled by our tools (modeling API, and parsers). Minor simplifications have been performed on annotations (see Chapter 12); currently, decision variables can be handled by our tools (modeling API, and parsers).

• XCSP3, version 3.0.4. Published on August 20, 2016. A few modifications about soft constraints have been achieved; see Chapter 9 and Section 11.3.1. Simple relaxation is now possible, simplified use of cost variable is authorized, and we have a better integration between relaxed constraints and cost functions. Constraint sum now admits variable coefficients. The semantics of constraints circuit, path and tree have been slightly modified.

• XCSP3, version 3.0.3. Published on May 20, 2016. Solutions can now be represented; see Section 2.11. Related to solutions, the constraint instantiation is introduced; see Section 4.1.8.2. It generalizes and replaces the constraint cube. XCSP3-core has been slightly updated: it involves now 20 constraints (including the constraint instantiation and the meta-constraint slide); see Section 1.5. More examples are given in Section 1.3, when presenting the skeleton of XCSP3 instances. A problem with the semantics of slide has been fixed; see Section 8.1.
• XCSP3, version 3.0.2. Published on February 5, 2016. XCSP3 is an intermediate format, as explained in Section 1.1. More information about “XML versus JSON” is given in Appendix D as well as in Section 1.4 of Chapter 1; limitations of JSON are identified. New universal attributes are introduced: note for associating a short comment with any element, and class for associating either predefined or user-defined tags with any element; this is discussed in Sections 1.7 and 10.2.

• XCSP3, version 3.0.1. Published on October 27, 2015. Converting XCSP3 instances from XML to JSON is discussed in a new Appendix as well as in Section 1.4 of Chapter 1. XCSP3-core is introduced in Section 1.5 of Chapter 1.

• XCSP3, version 3.0.0. First official version. Published on September 1, 2015.

• XCSP3, Release Candidate. Published on June 22, 2015.
Bibliography


